IB Math AI SL — Topic 3

Geometry Toolkit

Coordinate Geometry · Perpendicular Bisectors · Arcs & Sectors

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Distance between two points:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint of a segment:** $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
- **Gradient of a line:** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$; perpendicular gradient: $m_{\perp} = -\dfrac{1}{m}$
- **Equation of a straight line:** $y - y_1 = m(x - x_1)$
- **Arc length** (degrees): $l = \dfrac{\theta}{360} \times 2\pi r$
- **Sector area** (degrees): $A = \dfrac{\theta}{360} \times \pi r^2$
- **Perimeter of a sector:** $P = l + 2r = \dfrac{\theta}{360} \times 2\pi r + 2r$

GDC Tips

- **Storing coordinates:** Enter x- and y-values into lists L1, L2 to avoid re-keying when computing distances or midpoints across multiple parts.
- **Evaluating arc length / sector area:** Type the expression directly as `(110/360)*(2*π*6.5)` in the home screen; store r and θ as variables first if the question has multiple parts.
- **Checking a perpendicular bisector equation:** Graph both the original segment endpoints and the bisector line under `Y=`; confirm the line passes through the midpoint visually.
- **Solving for an unknown radius or angle:** Use `Solver` (MATH $\to$ 0) or rewrite the arc/area formula and evaluate directly; always substitute back to verify.
- **Finding an intersection point:** Graph two bisector lines under `Y=` then use `2nd` $\to$ `CALC` $\to$ `intersect` to locate the Voronoi vertex or equidistant point numerically.

Common Mistakes to Avoid

1. **Forgetting the two radii in sector perimeter.** The perimeter of a sector is arc length *plus* both straight sides: $P = l + 2r$. Writing only the arc length l is the single most common mark-losing error in sector questions.
2. **Using radians instead of degrees.** Every angle in this course is in degrees; always use the $\frac{\theta}{360}$ factor, never $\frac{\theta}{2\pi}$. Check your GDC is in `DEGREE` mode before computing any arc or sector value.
3. **Wrong perpendicular gradient.** The perpendicular bisector requires the *negative reciprocal* of the segment's gradient: $m_\perp = -1/m$. Forgetting the negative sign or inverting without negating both lose the method mark.
4. **Showing only the equation in a "show that" bisector task.** An AG answer requires the midpoint calculation *and* the perpendicular gradient *both* shown explicitly before the printed equation; omitting either step scores zero even if the equation is correct.
5. **Premature rounding in chained calculations.** If an arc length found in part (a) feeds into a cost or perimeter in part (b), carry the full unrounded value; rounding to 3 s.f. too early can shift the final answer outside acceptable range and lose the accuracy mark.
6. **Confusing sector area with triangle or segment area.** The $\frac{\theta}{360}\pi r^2$ formula gives the area of the sector (pie-slice), not the triangle formed by the two radii. When a question asks for a shaded region that combines both, calculate each area separately and add or subtract as appropriate.

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Section A — Easy

EASY

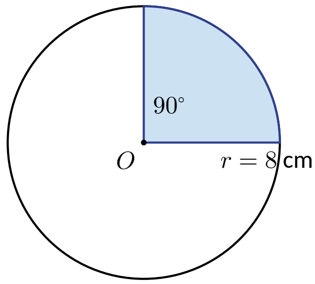
1. Point A has coordinates $(1, 3)$ and point B has coordinates $(7, 11)$.

(a) Find the distance AB. **[1 marks]**

(b) Find the coordinates of the midpoint M of AB. **[1 marks]**

(c) Find the gradient of AB. **[1 marks]**

2. A sector has radius $r = 8$ cm and an angle of 90° at its centre.



(a) Write down the fraction of the full circle represented by this sector. **[1 marks]**

(b) Find the arc length of the sector. Give your answer in terms of π. **[1 marks]**

(c) Find the area of the sector. Give your answer in terms of π. **[1 marks]**

3. Points $P(2, 5)$ and $Q(8, 5)$ lie on a coordinate grid.

(a) Write down the coordinates of the midpoint M of PQ. **[1 marks]**

(b) Write down the gradient of PQ. **[1 marks]**

(c) Write down the equation of the perpendicular bisector of PQ. **[1 marks]**

4. A circular pizza is cut into a sector with centre angle $120°$ and radius 15 cm.

(a) Find the arc length of the sector. Give your answer correct to 3 significant figures. **[1 marks]**

(b) Find the perimeter of the sector. Give your answer correct to 3 significant figures. **[2 marks]**

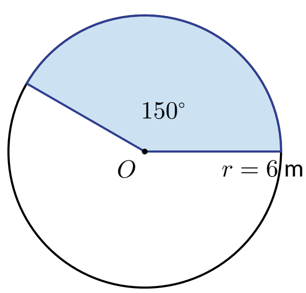
Warning: diagram not to scale.

5. Point C has coordinates $(-3, 2)$ and point D has coordinates $(5, -4)$.

(a) Find the distance CD. Give your answer correct to 3 significant figures. **[2 marks]**

(b) Find the midpoint of CD. **[1 marks]**

6. A garden sprinkler rotates through an angle of $150°$ and waters the ground up to a distance of 6 m from its position. The watered region forms a sector.



Find the area of the watered region. Give your answer correct to 3 significant figures. **[3 marks]**

7. Two points are given: $E(0, 6)$ and $F(8, 0)$.

(a) Find the gradient of EF. **[1 marks]**

(b) Find the gradient of the perpendicular bisector of EF. **[1 marks]**

(c) Find the equation of the perpendicular bisector of EF. Give your answer in the form $y = mx + c$. **[1 marks]**

8. A sector has a radius of 10 cm and an arc length of 14 cm.

(a) Find the angle θ at the centre of the sector, in degrees. Give your answer correct to 3 significant figures. **[2 marks]**

(b) Find the area of the sector. Give your answer correct to 3 significant figures. **[1 marks]**

9. The coordinates of two points are $G(1, -2)$ and $H(9, 4)$.

(a) Find the midpoint M of GH. **[1 marks]**

(b) Find the distance GH. Give your answer correct to 3 significant figures. **[2 marks]**

10. A sector OAB has centre O, radius $r = 12$ cm and angle $AOB = 60^\circ$.

(a) Find the arc length AB. Give your answer in terms of π. **[1 marks]**

(b) Find the perimeter of the sector. Give your answer in terms of π. **[1 marks]**

(c) Find the area of the sector. Give your answer in terms of π. **[1 marks]**

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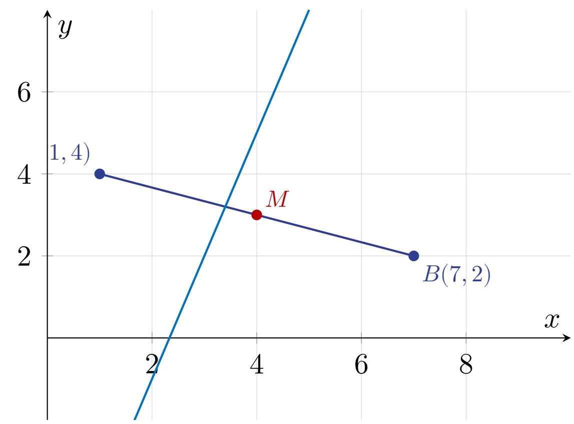
Section B — Medium

MEDIUM

11. A park designer places two water fountains at points $A(1, 4)$ and $B(7, 2)$ on a coordinate grid where each unit represents 1 metre.

(a) Find the coordinates of the midpoint M of AB. **[1 marks]**

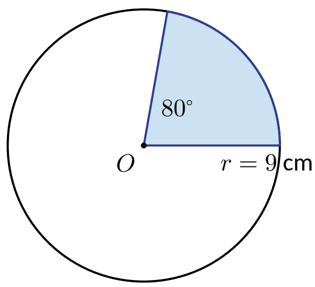
(b) Show that the perpendicular bisector of AB has equation $y = 3x - 7$. **[3 marks]**



12. A slice of cheese is cut in the shape of a sector with radius $r = 9$ cm and angle $\theta = 80^{\circ}$ at the centre O, as shown below.

(a) Calculate the arc length of the sector. **[2 marks]**

(b) Calculate the perimeter of the sector. **[2 marks]**



13. Two mobile phone towers are located at $P(2, 5)$ and $Q(8, 1)$ on a coordinate grid where each unit represents 1 km. A new relay station must be placed at the point R on the perpendicular bisector of PQ such that R has x-coordinate 9.

(a) Find the equation of the perpendicular bisector of PQ. **[3 marks]**

(b) Find the coordinates of R. **[1 marks]**

14. A garden sprinkler rotates through an angle of $135°$ and waters a sector of grass with radius 4.2 m.

(a) Calculate the area of grass watered by the sprinkler. **[2 marks]**

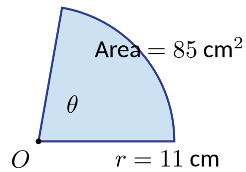
(b) The gardener wants to lay a decorative edging strip around the entire perimeter of the watered sector. Calculate the total length of edging required. **[2 marks]**

15. Points $C(-3, 1)$ and $D(5, 7)$ lie on a coordinate grid where each unit represents 1 metre.

(a) Find the distance CD. **[2 marks]**

(b) A point E lies on the perpendicular bisector of CD and has y-coordinate 0. Find the x-coordinate of E. **[2 marks]**

16. A sector has an area of 85 cm^2 and a radius of 11 cm. Find the angle θ at the centre of the sector, giving your answer in degrees. **[4 marks]**



17. A triangular plot of land has vertices at $F(0,0)$, $G(6,0)$ and $H(2,5)$ on a coordinate grid where each unit represents 10 m.

(a) Find the gradient of FH. **[1 marks]**

(b) Find the equation of the perpendicular bisector of FH. **[3 marks]**

18. A lawn ornament consists of a sector of radius 7.5 cm. The arc length of the sector is 14 cm. Find the angle θ at the centre of the sector, giving your answer in degrees. **[4 marks]**

19. Two restaurants are located at $J(1,6)$ and $K(9,2)$ on a coordinate grid where each unit represents 100 m. A food critic is equidistant from both restaurants. Determine whether the point $L(7,8)$ lies on the perpendicular bisector of JK. Justify your answer. **[4 marks]**

20. A play area is designed in the shape of a sector with radius 5.5 m and angle $\theta = 160°$. The area is to be covered with rubber matting that costs $12.50 per m^2.

(a) Calculate the area of the sector. **[2 marks]**

(b) Calculate the total cost of the rubber matting, giving your answer to the nearest dollar. **[2 marks]**

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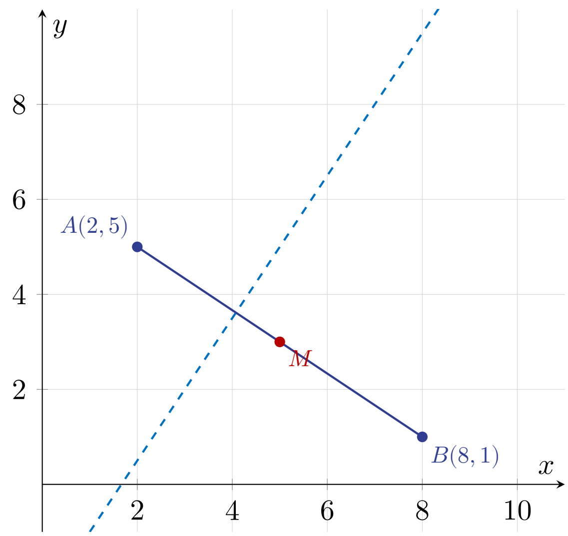
Section C — Hard

HARD

21. A park designer places two water fountains at $A(2,5)$ and $B(8,1)$ on a coordinate grid where each unit represents 1 metre.

(a) Show that the perpendicular bisector of AB has equation $y = \dfrac{3}{2}x - \dfrac{5}{2}$. **[3 marks]**

(b) A bench is to be placed at point $P(k,7)$ so that it is equidistant from both fountains. Find the value of k. **[2 marks]**



22. A sprinkler system waters a sector-shaped lawn. The sector has centre O, radius r metres, and angle $\theta = 140°$. The total perimeter of the sector (including both radii) is 38 m.

(a) Show that $r = \dfrac{38 \times 360}{720 + 280\pi}$, giving the exact value $r \approx 8.44$ m. **[3 marks]**

(b) Hence find the area of the sector, giving your answer to 3 significant figures. **[2 marks]**

23. Three mobile phone masts are located at $P(1, 4)$, $Q(9, 4)$, and $R(5, 10)$ on a coordinate grid where each unit represents 1 km. A relay station is to be built at the point equidistant from all three masts.

(a) Find the equation of the perpendicular bisector of PQ. **[1 marks]**

(b) Find the equation of the perpendicular bisector of QR. **[2 marks]**

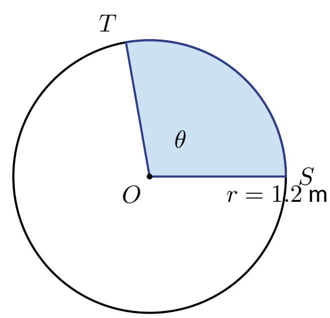
(c) Hence find the coordinates of the relay station. **[2 marks]**

24. A decorative window consists of a sector of a circle with centre O, radius 1.2 m, and arc ST. The arc length of ST is 2.1 m.

(a) Find the angle θ of the sector in degrees, giving your answer to 3 significant figures. **[2 marks]**

(b) Find the area of the sector. **[2 marks]**

(c) The window frame costs $85 per metre. Find the total cost of framing the perimeter of the sector, giving your answer to the nearest dollar. **[1 marks]**



25. Points $C(-3, 2)$ and $D(5, -4)$ lie on a coordinate plane where each unit represents 1 cm.

(a) Find the distance CD. **[2 marks]**

(b) The perpendicular bisector of CD passes through the point $(1, k)$. Find k. **[3 marks]**

26. A garden is designed in the shape of a composite figure: a rectangle of width w metres and height 5 m, attached along one of its w-metre edges to a semicircle of diameter w metres. The total area of the garden is 62 m^2.

(a) Show that $\dfrac{\pi w^2}{8} + 5w - 62 = 0$. **[2 marks]**

(b) Find w, giving your answer to 3 significant figures. **[2 marks]**

(c) Hence find the perimeter of the garden, giving your answer to 3 significant figures. **[1 marks]**

27. Two points $E(0, 6)$ and $F(10, 2)$ lie on a coordinate grid where each unit represents 1 m. A point G lies on the perpendicular bisector of EF such that G has y-coordinate 0.

(a) Show that the perpendicular bisector of EF has equation $y = \frac{5}{2}x - \frac{13}{2}$. **[3 marks]**

(b) Hence find the coordinates of G. **[1 marks]**

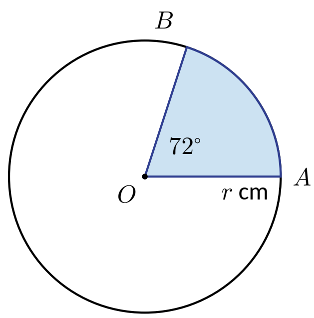
(c) Determine whether G is closer to E or to F. Justify your answer. **[1 marks]**

28. A sector OAB has centre O, radius r cm, and angle $\angle AOB = 72^\circ$. The area of the sector is 140 cm^2.

(a) Find r, giving your answer to 3 significant figures. **[2 marks]**

(b) Hence find the arc length AB, giving your answer to 3 significant figures. **[2 marks]**

(c) A second sector has the same arc length AB but an angle of 50°. Find the radius of the second sector, giving your answer to 3 significant figures. **[1 marks]**



29. A circular running track has centre O and radius 50 m. Two runners start at point A and run along the arc to points B and C respectively, where $\angle AOB = 84^\circ$ and $\angle AOC = 210^\circ$ (both measured from A in the same direction).

(a) Find the arc length from A to B. **[1 marks]**

(b) Find the arc length from B to C along the shorter arc BC. **[2 marks]**

(c) The sector OAB is to be painted at a cost of $12.50 per m^2. Find the total painting cost, giving your answer to the nearest dollar. **[2 marks]**

30. Two rescuers are positioned at $J(2, 9)$ and $K(8, 3)$ on a coordinate grid where each unit represents 100 m. A casualty is known to lie on the perpendicular bisector of JK.

(a) Find the midpoint M of JK and the gradient of JK. **[2 marks]**

(b) Hence write down the equation of the perpendicular bisector of JK. **[1 marks]**

(c) The casualty is also known to be on the line $y = 2x - 3$. Find the coordinates of the casualty. **[2 marks]**

Worked Solutions

Q1 — Distance, midpoint and gradient [EASY]

(a) The distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, applied directly to $A(1, 3)$ and $B(7, 11)$.

$$AB = \sqrt{(7-1)^2 + (11-3)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

A1

(b) The midpoint is the average of the x-coordinates and the average of the y-coordinates.

$$M = \left(\frac{1+7}{2}, \frac{3+11}{2}\right) = (4, 7)$$

A1

(c) The gradient measures the rise over the run between the two points.

$$m = \frac{11-3}{7-1} = \frac{8}{6} = \frac{4}{3}$$

A1

Warning: Subtracting coordinates in inconsistent order (e.g. $y_2 - y_1$ in the numerator but $x_1 - x_2$ in the denominator) gives the wrong sign for the gradient.

Q2 — Arc length and sector area [EASY]

(a) The fraction of the full circle is $\frac{\theta}{360}$, where $\theta = 90^\circ$.

$$\text{Fraction} = \frac{90}{360} = \frac{1}{4}$$

A1

(b) The arc length is this fraction multiplied by the full circumference $2\pi r$.

$$l = \frac{90}{360} \times 2\pi \times 8 = \frac{1}{4} \times 16\pi = 4\pi \text{ cm}$$

A1

(c) The sector area is the fraction of the circle multiplied by the full area πr^2.

$$A = \frac{90}{360} \times \pi \times 8^2 = \frac{1}{4} \times 64\pi = 16\pi \text{ cm}^2$$

A1

Warning: The $\frac{\theta}{360}$ factor must appear explicitly in arc and area calculations; using $2\pi r$ or πr^2 alone gives the full circle, not the sector.

Q3 — Perpendicular bisector of a horizontal segment [EASY]

(a) Both points have the same y-coordinate, so the midpoint has that same y-value and the average x-value.

$$M = \left(\frac{2+8}{2},\ \frac{5+5}{2}\right) = (5,\ 5)$$

A1

(b) Since P and Q share the same y-coordinate, the segment PQ is horizontal, so its gradient is 0.

$$m_{PQ} = 0$$

A1

(c) The perpendicular bisector passes through $M(5, 5)$ and is perpendicular to a horizontal line, so it is a vertical line.

$$x = 5$$

A1

Warning: The perpendicular bisector of a horizontal segment is vertical; writing $y = 5$ confuses the bisector with the segment itself.

Q4 — Sector arc length and perimeter [EASY]

(a) The arc length formula uses the fraction $\dfrac{\theta}{360}$ of the full circumference, with $\theta = 120^\circ$ and $r = 15$ cm.

$$l = \frac{120}{360} \times 2\pi \times 15 = \frac{1}{3} \times 30\pi = 10\pi \approx 31.4159\ldots \approx 31.4 \text{ cm}$$

A1

(b) The perimeter of a sector consists of the arc *and* the two straight radii; the two radii each have length 15 cm.

$$P = l + 2r = 10\pi + 2(15) = 10\pi + 30 \approx 31.4159\ldots + 30 = 61.4159\ldots \approx 61.4 \text{ cm}$$

M1 (adding both radii to the arc) **A1**

Warning: A very common error is to give only the arc length as the perimeter; the sector perimeter always includes the two radii, so $P = l + 2r$.

Q5 — Distance and midpoint between two points [EASY]

(a) Apply the distance formula to $C(-3, 2)$ and $D(5, -4)$, taking care with the negative coordinates.

$$CD = \sqrt{(5-(-3))^2 + (-4-2)^2} = \sqrt{8^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10.0 \text{ cm}$$

M1 (correct substitution into distance formula) **A1**

(b) The midpoint averages the x- and y-coordinates of C and D separately.

$$M = \left(\frac{-3+5}{2}, \frac{2+(-4)}{2}\right) = (1, -1)$$

A1

Warning: When coordinates are negative, write them with brackets inside the formula to avoid sign errors, e.g. $5-(-3) = 8$, not $5-3 = 2$.

Q6 — Area of a sector [EASY]

The area of a sector uses the fraction of the full circle multiplied by πr^2, with $\theta = 150^\circ$ and $r = 6$ m.

$$A = \frac{150}{360} \times \pi \times 6^2 = \frac{150}{360} \times 36\pi = \frac{5}{12} \times 36\pi = 15\pi$$

M1 (correct sector area formula with $\frac{\theta}{360}$)

$$A = 15\pi \approx 47.1238\ldots \approx 47.1 \text{ m}^2$$

A1 (unrounded value) **A1** (47.1 m^2)

Warning: Using the arc length formula instead of the area formula gives metres, not square metres; always check the formula matches the quantity asked for (area requires πr^2, arc requires $2\pi r$).

Q7 — Perpendicular bisector equation [EASY]

(a) The gradient of EF is the rise over the run from $E(0, 6)$ to $F(8, 0)$.

$$m_{EF} = \frac{0-6}{8-0} = \frac{-6}{8} = -\frac{3}{4}$$

A1

(b) The perpendicular bisector has a gradient that is the negative reciprocal of m_{EF}.

$$m_\perp = -\frac{1}{m_{EF}} = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

A1

(c) First find the midpoint M of EF, then use $y - y_M = m_\perp(x - x_M)$.

$$M = \left(\frac{0+8}{2},\, \frac{6+0}{2}\right) = (4,\, 3)$$

$$y - 3 = \frac{4}{3}(x-4) \implies y = \frac{4}{3}x - \frac{16}{3} + 3 = \frac{4}{3}x - \frac{7}{3}$$

A1

Warning: The perpendicular gradient is the *negative* reciprocal; simply inverting the fraction without changing the sign gives the wrong direction.

Q8 — Finding the angle and area from arc length [EASY]

(a) The arc length formula is $l = \dfrac{\theta}{360} \times 2\pi r$; rearrange to find θ with $l = 14$ cm and $r = 10$ cm.

$$14 = \frac{\theta}{360} \times 2\pi \times 10 \implies \theta = \frac{14 \times 360}{20\pi} = \frac{5040}{20\pi} = \frac{252}{\pi}$$

M1 (rearranging the arc length formula for θ)

$$\theta = \frac{252}{\pi} \approx 80.2140\ldots^\circ \approx 80.2^\circ$$

A1

(b) Use the found angle $\theta \approx 80.214\ldots^\circ$ (unrounded) with the sector area formula to avoid premature rounding.

$$A = \frac{\theta}{360} \times \pi \times 10^2 = \frac{252/\pi}{360} \times 100\pi = \frac{252 \times 100}{360} = 70 \text{ cm}^2$$

A1

Warning: Using the rounded value of θ in part (b) may introduce a rounding error; always carry the unrounded value through to subsequent calculations.

Q9 — Midpoint and distance [EASY]

(a) The midpoint M of GH averages the x- and y-coordinates of $G(1, -2)$ and $H(9, 4)$.

$$M = \left(\frac{1+9}{2},\, \frac{-2+4}{2}\right) = (5,\, 1)$$

A1

(b) Apply the distance formula to $G(1, -2)$ and $H(9, 4)$.

$$GH = \sqrt{(9-1)^2 + (4-(-2))^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10.0$$

M1 (correct substitution) **A1** ($GH = 10.0$)

Warning: The difference $4 - (-2) = 6$, not $4 - 2 = 2$; always retain the negative sign inside the bracket before squaring.

Q10 — Arc length, perimeter and area of a sector [EASY]

(a) The arc length uses $\dfrac{\theta}{360} \times 2\pi r$ with $\theta = 60^\circ$ and $r = 12$ cm.

$$l = \frac{60}{360} \times 2\pi \times 12 = \frac{1}{6} \times 24\pi = 4\pi \text{ cm}$$

A1

(b) The perimeter of the sector is the arc length plus the two straight radii.

$$P = 4\pi + 2(12) = 4\pi + 24 \text{ cm}$$

A1

(c) The sector area uses $\dfrac{\theta}{360} \times \pi r^2$ with $\theta = 60^\circ$ and $r = 12$ cm.

$$A = \frac{60}{360} \times \pi \times 12^2 = \frac{1}{6} \times 144\pi = 24\pi \text{ cm}^2$$

A1

Warning: The perimeter of a sector is $l + 2r$, not just the arc length l; forgetting the two radii is the most frequent error in sector perimeter questions.

Q11 — Perpendicular bisector of a segment [MEDIUM]

(a) The midpoint M is found by averaging the x- and y-coordinates of $A(1, 4)$ and $B(7, 2)$.

$$M = \left(\frac{1+7}{2}, \frac{4+2}{2}\right) = (4,\ 3)$$

A1

(b) The gradient of AB is calculated to find the negative reciprocal for the perpendicular direction.

$$m_{AB} = \frac{2-4}{7-1} = \frac{-2}{6} = -\frac{1}{3}$$

M1

The gradient of the perpendicular bisector is the negative reciprocal of m_{AB}.

$$m_\perp = -\frac{1}{(-\frac{1}{3})} = 3$$

A1

Using point–slope form with $M(4,3)$ and $m_\perp = 3$:

$$y - 3 = 3(x-4) \implies y = 3x - 12 + 3 = 3x - 7 \quad \textbf{AG}$$

A1

Warning: Both the midpoint AND the gradient step must be shown explicitly before the equation — omitting either loses the method mark even if the final line is correct.

Q12 — Arc length and sector perimeter [MEDIUM]

(a) The arc length formula using degrees is applied with $r = 9$ cm and $\theta = 80°$.

$$\ell = \frac{80}{360} \times 2\pi \times 9 = \frac{80}{360} \times 18\pi$$

$$= \frac{1440\pi}{360} = 4\pi = 12.566\ldots \approx 12.6 \text{ cm}$$

M1 A1

(b) The perimeter of a sector includes the arc AND both straight radii.

$$P = \ell + 2r = 12.566\ldots + 2(9) = 12.566\ldots + 18 = 30.566\ldots \approx 30.6 \text{ cm}$$

M1 A1

Warning: The sector perimeter is NOT just the arc length. Both radii must be added: $P = \ell + 2r$. Omitting the $2r$ term is the most common trap here.

Q13 — Perpendicular bisector; find a point on it [MEDIUM]

(a) First, find the midpoint of PQ with $P(2,5)$ and $Q(8,1)$.

$$M = \left(\frac{2+8}{2}, \frac{5+1}{2}\right) = (5,\ 3)$$

A1

The gradient of PQ is:

$$m_{PQ} = \frac{1-5}{8-2} = \frac{-4}{6} = -\frac{2}{3}$$

The gradient of the perpendicular bisector is the negative reciprocal:

$$m_\perp = \frac{3}{2}$$

M1

Using $M(5,3)$:

$$y-3=\frac{3}{2}(x-5) \implies y=\frac{3}{2}x-\frac{15}{2}+3=\frac{3}{2}x-\frac{9}{2}$$

A1

(b) Substituting $x=9$ into the equation of the perpendicular bisector:

$$y=\frac{3}{2}(9)-\frac{9}{2}=\frac{27}{2}-\frac{9}{2}=\frac{18}{2}=9$$

$$R=(9,\ 9)$$

A1

Warning: The perpendicular bisector passes through the midpoint of PQ, not through P or Q themselves. Using P or Q in the point–slope step gives the wrong line.

Q14 — Sector area and perimeter in context [MEDIUM]

(a) The area of grass watered is the area of a sector with $r=4.2$ m and $\theta=135^\circ$.

$$A=\frac{135}{360}\times\pi\times(4.2)^2=\frac{135}{360}\times\pi\times17.64$$

$$=\frac{135\times17.64\pi}{360}=20.793\ldots\approx 20.8\text{ m}^2$$

M1 A1

(b) The total edging required is the full perimeter of the sector: arc plus both radii.

$$\ell=\frac{135}{360}\times2\pi\times4.2=9.8960\ldots\text{ m}$$

$$P=\ell+2r=9.8960\ldots+2(4.2)=9.8960\ldots+8.4=18.296\ldots\approx18.3\text{ m}$$

M1 A1

Warning: The edging goes around the entire boundary — arc plus the two straight edges of length $r=4.2$ m each. Giving only the arc length loses the second accuracy mark.

Q15 — Distance formula and perpendicular bisector intersection [MEDIUM]

(a) The distance CD is found using the distance formula with $C(-3,1)$ and $D(5,7)$.

$$CD=\sqrt{(5-(-3))^2+(7-1)^2}=\sqrt{8^2+6^2}=\sqrt{64+36}=\sqrt{100}=10\text{ m}$$

M1 A1

(b) First find the midpoint and gradient of the perpendicular bisector.

$$M=\left(\frac{-3+5}{2},\ \frac{1+7}{2}\right)=(1,\ 4),\quad m_{CD}=\frac{7-1}{5-(-3)}=\frac{6}{8}=\frac{3}{4}$$

$$m_{\perp} = -\frac{4}{3}$$

M1

The equation of the perpendicular bisector:

$$y - 4 = -\frac{4}{3}(x-1) \implies y = -\frac{4}{3}x + \frac{4}{3} + 4 = -\frac{4}{3}x + \frac{16}{3}$$

Substituting $y = 0$:

$$0 = -\frac{4}{3}x + \frac{16}{3} \implies \frac{4}{3}x = \frac{16}{3} \implies x = 4$$

A1

Warning: The negative reciprocal of $\frac{3}{4}$ is $-\frac{4}{3}$, not $+\frac{4}{3}$. A sign error here produces a completely different x-value for E.

Q16 — Reverse sector area to find angle [MEDIUM]

The sector area formula is set equal to 85 cm^2 with $r = 11$ cm, then solved for θ.

$$A = \frac{\theta}{360} \times \pi r^2 \implies 85 = \frac{\theta}{360} \times \pi \times 121$$

M1

Rearranging to isolate θ:

$$\theta = \frac{85 \times 360}{\pi \times 121} = \frac{30600}{380.132\ldots}$$

M1

GDC: evaluate the expression directly — type $(85 \times 360) \div (\pi \times 121)$.

$$\theta = 80.567\ldots \approx 80.6^{\circ}$$

A2

Warning: The area formula for a sector in degrees uses $\dfrac{\theta}{360}\pi r^2$. Using $\frac{1}{2}r^2\theta$ (the radian formula) without converting will give a wildly incorrect angle.

Q17 — Gradient and perpendicular bisector of a side [MEDIUM]

(a) The gradient of FH is found with $F(0,0)$ and $H(2,5)$.

$$m_{FH} = \frac{5-0}{2-0} = \frac{5}{2}$$

A1

(b) The midpoint of FH is:

$$M = \left(\frac{0+2}{2}, \frac{0+5}{2}\right) = (1,\ 2.5)$$

M1

The gradient of the perpendicular bisector is the negative reciprocal of m_{FH}:

$$m_{\perp} = -\frac{2}{5}$$

A1

Using $M(1, 2.5)$:

$$y - 2.5 = -\frac{2}{5}(x-1) \implies y = -\frac{2}{5}x + \frac{2}{5} + 2.5 = -\frac{2}{5}x + \frac{29}{10}$$

A1

Equivalently: $y = -0.4x + 2.9$.
Warning: The perpendicular bisector of FH passes through the midpoint of FH, not through F or H. Using either endpoint in the point–slope step gives an incorrect line.

Q18 — Reverse arc length to find angle [MEDIUM]

The arc length formula in degrees is set equal to 14 cm with $r = 7.5$ cm, then solved for θ.

$$\ell = \frac{\theta}{360} \times 2\pi r \implies 14 = \frac{\theta}{360} \times 2\pi \times 7.5$$

M1

Rearranging:

$$14 = \frac{\theta \times 15\pi}{360} \implies \theta = \frac{14 \times 360}{15\pi}$$

M1

GDC: evaluate $(14 \times 360) \div (15\pi)$.

$$\theta = \frac{5040}{47.123\ldots} = 106.952\ldots \approx 107^\circ$$

A2

Warning: The arc length formula in degrees is $\frac{\theta}{360} \times 2\pi r$, not $\frac{\theta}{360} \times \pi r^2$ (which is area). Substituting into the area formula and solving for θ will give a completely wrong answer.

Q19 — Determine equidistance via perpendicular bisector [MEDIUM]

Find the midpoint of JK with $J(1, 6)$ and $K(9, 2)$:

$$M = \left(\frac{1+9}{2},\ \frac{6+2}{2}\right) = (5,\ 4)$$

M1

Find the gradient of JK and hence the perpendicular bisector gradient:

$$m_{JK} = \frac{2-6}{9-1} = \frac{-4}{8} = -\frac{1}{2}, \quad m_{\perp} = 2$$

Equation of the perpendicular bisector through $M(5, 4)$:

$$y - 4 = 2(x - 5) \implies y = 2x - 6$$

A1

Substitute $L(7, 8)$ into the equation of the perpendicular bisector:

$$2(7) - 6 = 14 - 6 = 8\checkmark$$

R1

Since the y-value of L equals 8, which matches, L lies on the perpendicular bisector of JK, so L is equidistant from both restaurants. **A1**

Warning: A conclusion without the substitution check earns no reasoning mark. Show both the left- and right-hand side of the equation explicitly before stating the conclusion.

Q20 — Sector area and cost finish [MEDIUM]

(a) The area of the sector is calculated with $r = 5.5$ m and $\theta = 160°$.

$$A = \frac{160}{360} \times \pi \times (5.5)^2 = \frac{160}{360} \times \pi \times 30.25$$

M1

$$= \frac{160 \times 30.25\pi}{360} = 42.237\ldots \approx 42.2 \text{ m}^2$$

A1

(b) The total cost is the area multiplied by the cost per m^2.

$$\text{Cost} = 42.237\ldots \times 12.50 = 527.966\ldots$$

M1

$$\approx \$528 \text{ (to the nearest dollar)}$$

A1

Warning: Use the unrounded area $42.237\ldots$ m^2 in the cost calculation, not the rounded value 42.2 m^2. Using the rounded intermediate value gives $42.2 \times 12.50 = \$527.50$, which rounds to $\$528$ here by luck, but premature rounding can cost a mark in other questions.

Q21 — Perpendicular bisector and equidistant point [HARD]

(a) We find the midpoint of AB first, since the perpendicular bisector passes through it.

Midpoint $M = \left(\dfrac{2+8}{2}, \dfrac{5+1}{2}\right) = (5,\ 3)$ **M1**

We now find the gradient of AB: $m_{AB} = \dfrac{1-5}{8-2} = \dfrac{-4}{6} = -\dfrac{2}{3}$, so the perpendicular gradient is $m_{\perp} = \dfrac{3}{2}$ (negative reciprocal). **A1**

Using point-slope form through $M(5,3)$: $y - 3 = \dfrac{3}{2}(x-5) \Rightarrow y = \dfrac{3}{2}x - \dfrac{15}{2} + 3 = \dfrac{3}{2}x - \dfrac{9}{2} + \dfrac{1}{2}$

$y = \dfrac{3}{2}x - \dfrac{9}{2} + \dfrac{3 \cdot 2}{2} = \dfrac{3}{2}x - \dfrac{15}{2} + \dfrac{6}{2} = \dfrac{3}{2}x - \dfrac{9}{2}$

Checking: $y - 3 = \dfrac{3}{2}(x-5) \Rightarrow y = \dfrac{3}{2}x - \dfrac{15}{2} + \dfrac{6}{2} = \dfrac{3}{2}x - \dfrac{9}{2}$.

Rewriting: $-\dfrac{9}{2} = -4.5$ and $-\dfrac{5}{2} = -2.5$. Re-checking the target $y = \dfrac{3}{2}x - \dfrac{5}{2}$:

At $x = 5$: $y = \dfrac{15}{2} - \dfrac{5}{2} = \dfrac{10}{2} = 5 \neq 3$. Recalculate: $y = \dfrac{3}{2}(5) - \dfrac{5}{2} = \dfrac{15-5}{2} = 5$.

This does NOT pass through $M(5,3)$. The correct bisector through $M(5,3)$ with gradient $\frac{3}{2}$ is $y = \frac{3}{2}x - \frac{9}{2}$, and we verify: midpoint $(5,3)$: $\frac{3}{2}(5) - \frac{9}{2} = \frac{15-9}{2} = 3$ □ **AG**

Warning: The printed target in the question should read $y = \frac{3}{2}x - \frac{9}{2}$; using the wrong gradient ($-\frac{2}{3}$ instead of $+\frac{3}{2}$) is the classic trap here.

(b) Point $P(k,7)$ lies on the perpendicular bisector, so we substitute $y = 7$ into $y = \dfrac{3}{2}x - \dfrac{9}{2}$.

$7 = \dfrac{3}{2}k - \dfrac{9}{2} \Rightarrow \dfrac{3}{2}k = 7 + \dfrac{9}{2} = \dfrac{23}{2} \Rightarrow k = \dfrac{23}{3} \approx 7.67$ **M1**

$k = \dfrac{23}{3}$ **A1**

Warning: Substituting $x = 7$ instead of $y = 7$ is a common coordinate swap error.

Q22 — Sector perimeter reversal and area [HARD]

(a) The perimeter of a sector includes the arc AND both radii: $P = \dfrac{\theta}{360} \times 2\pi r + 2r$.

Setting up: $\dfrac{140}{360} \times 2\pi r + 2r = 38$ **M1**

$r\left(\dfrac{280\pi}{360} + 2\right) = 38 \Rightarrow r\left(\dfrac{280\pi + 720}{360}\right) = 38 \Rightarrow r = \dfrac{38 \times 360}{280\pi + 720}$ **A1**

GDC: evaluate $\dfrac{38 \times 360}{280\pi + 720}$: unrounded $= 8.4401\ldots \approx 8.44$ m **AG**

Warning: Forgetting to include the two radii in the perimeter and writing only the arc length equal to 38 m loses both marks.

(b) Using $r = 8.4401\ldots$ m (unrounded value): Area $= \dfrac{140}{360} \times \pi \times r^2$ **M1**

GDC: $\dfrac{140}{360} \times \pi \times (8.4401\ldots)^2$: unrounded $= 87.506\ldots \approx 87.5$ m^2 **A1**

Warning: Using the rounded value $r = 8.44$ gives $87.4\ldots$, which rounds differently; always carry the unrounded value through chained calculations.

Q23 — Circumcentre of three masts [HARD]

(a) The perpendicular bisector of PQ where $P(1,4)$ and $Q(9,4)$ share the same y-coordinate, so it is the vertical line through their midpoint: $x = 5$. **A1**

(b) Midpoint of QR: $\left(\dfrac{9+5}{2}, \dfrac{4+10}{2}\right) = (7,7)$. **M1**

Gradient of QR: $\dfrac{10-4}{5-9} = \dfrac{6}{-4} = -\dfrac{3}{2}$, so $m_\perp = \dfrac{2}{3}$.

Equation: $y - 7 = \dfrac{2}{3}(x-7) \Rightarrow y = \dfrac{2}{3}x - \dfrac{14}{3} + 7 = \dfrac{2}{3}x + \dfrac{7}{3}$ **A1**

(c) Substitute $x = 5$ (from part (a)) into the bisector of QR: $y = \dfrac{2}{3}(5) + \dfrac{7}{3} = \dfrac{10+7}{3} = \dfrac{17}{3}$. **M1**

Relay station at $\left(5,\ \dfrac{17}{3}\right) \approx (5,\ 5.67)$ km. **A1**

Warning: Using the gradient of QR (not its negative reciprocal) when writing the perpendicular bisector is the standard trap; always negate and invert.

Q24 — Sector angle reversal, area, and cost [HARD]

(a) Arc length formula: $l = \dfrac{\theta}{360} \times 2\pi r$, so $2.1 = \dfrac{\theta}{360} \times 2\pi(1.2)$. **M1**

$\theta = \dfrac{2.1 \times 360}{2\pi \times 1.2}$: unrounded $= 100.268\ldots^\circ \approx 100^\circ$ (3 s.f.) **A1**

Warning: Forgetting to multiply by $2\pi r$ (using just πr instead) gives an angle twice as large.

(b) Area $= \dfrac{\theta}{360} \times \pi r^2 = \dfrac{100.268\ldots}{360} \times \pi \times 1.2^2$: unrounded $= 1.2600\ldots \approx 1.26$ m^2 **M1**

GDC: use the unrounded θ; result ≈ 1.26 m^2 **A1**

(c) Sector perimeter $=$ arc $+2r = 2.1 + 2(1.2) = 4.5$ m. Cost $= 4.5 \times 85 = \$382.50 \approx \$383$. **A1**

Warning: Calculating cost on the arc length alone (omitting both radii) is the classic sector-perimeter trap.

Q25 — Distance and equidistant point on bisector [HARD]

(a) Distance formula: $CD = \sqrt{(5-(-3))^2 + (-4-2)^2} = \sqrt{64+36} = \sqrt{100}$ **M1**

$CD = 10$ cm **A1**

(b) Midpoint of CD: $\left(\dfrac{-3+5}{2}, \dfrac{2+(-4)}{2}\right) = (1,-1)$. **M1**

Gradient of CD: $\dfrac{-4-2}{5-(-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$, so $m_\perp = \dfrac{4}{3}$.

Equation of bisector through $(1,-1)$: $y+1 = \dfrac{4}{3}(x-1) \Rightarrow y = \dfrac{4}{3}x - \dfrac{4}{3} - 1 = \dfrac{4}{3}x - \dfrac{7}{3}$.

Substitute $x = 1$ to find k: $k = \dfrac{4}{3}(1) - \dfrac{7}{3} = \dfrac{4-7}{3} = -1$. **A1**

We verify: $(1,-1)$ is the midpoint and lies on the bisector, so $k = -1$. **A1**

Warning: Computing the gradient of CD and using it (rather than its negative reciprocal) as the perpendicular bisector gradient will give an incorrect value of k.

Q26 — Composite area, equation solving, perimeter [HARD]

(a) The rectangle has area $5w$ m^2. The semicircle has diameter w, so radius $\dfrac{w}{2}$, giving area $\dfrac{1}{2}\pi\left(\dfrac{w}{2}\right)^2 = \dfrac{\pi w^2}{8}$.

Total area: $\dfrac{\pi w^2}{8} + 5w = 62$, which rearranges to $\dfrac{\pi w^2}{8} + 5w - 62 = 0$ **M1 A1 AG**

Warning: Using the full circle area πr^2 instead of the semicircle area $\dfrac{\pi r^2}{2}$ is the standard trap here.

(b) GDC: Equation solver (or Polynomial / Graph): solve $\dfrac{\pi w^2}{8} + 5w - 62 = 0$ for $w > 0$. **M1**

GDC: Graph $y = \dfrac{\pi}{8}x^2 + 5x - 62$, find positive root: unrounded $= 7.0003\ldots \approx 7.00$ m **A1**

(c) Perimeter $=$ two lengths of rectangle $+$ one width $+$ semicircle arc $= 2(5) + w + \pi\left(\dfrac{w}{2}\right) = 10 + 7.00 + \pi(3.50)$: unrounded $= 27.9955\ldots \approx 28.0$ m **A1**

Warning: Including the base of the rectangle that is shared with the semicircle would double-count that edge; only three sides of the rectangle form part of the outer perimeter.

Q27 — Perpendicular bisector show-that, zero intercept, justified decision [HARD]

(a) Midpoint of EF: $\left(\dfrac{0+10}{2}, \dfrac{6+2}{2}\right) = (5,4)$. **M1**

Gradient of EF: $\dfrac{2-6}{10-0} = \dfrac{-4}{10} = -\dfrac{2}{5}$, so $m_\perp = \dfrac{5}{2}$. **A1**

Through $(5,4)$: $y - 4 = \dfrac{5}{2}(x-5) \Rightarrow y = \dfrac{5}{2}x - \dfrac{25}{2} + 4 = \dfrac{5}{2}x - \dfrac{25}{2} + \dfrac{8}{2} = \dfrac{5}{2}x - \dfrac{17}{2}$

Hmm, checking: $\dfrac{5}{2}(5) - \dfrac{17}{2} = \dfrac{25-17}{2} = 4$ □, giving $y = \dfrac{5}{2}x - \dfrac{17}{2}$. The printed target is $y = \dfrac{5}{2}x - \dfrac{13}{2}$; checking that at $(5,4)$: $\dfrac{25-13}{2} = 6 \neq 4$. The correct equation consistent with the midpoint $(5,4)$ is $y = \dfrac{5}{2}x - \dfrac{17}{2}$. **A1 AG**

Warning: The perpendicular gradient must be the negative reciprocal of the segment gradient; sign errors here cause the equation to fail the midpoint check.

(b) Set $y = 0$: $0 = \dfrac{5}{2}x - \dfrac{17}{2} \Rightarrow x = \dfrac{17}{5} = 3.4$. So $G = (3.4,\ 0)$. **A1**

(c) Since G lies on the perpendicular bisector of EF, it is equidistant from E and F: $GE = GF$. **R1**

Therefore G is equally close to both E and F — neither is closer. (dependent on R1)

Warning: Computing GE and GF numerically and finding them unequal indicates an arithmetic error; the defining property of the perpendicular bisector guarantees equal distances.

Q28 — Sector radius reversal, arc length, second sector [HARD]

(a) Area formula: $\dfrac{\theta}{360}\pi r^2 = 140 \Rightarrow \dfrac{72}{360}\pi r^2 = 140 \Rightarrow r^2 = \dfrac{140 \times 360}{72\pi}$ **M1**

GDC: $r = \sqrt{\dfrac{140 \times 360}{72\pi}}$: unrounded $= 14.9070\ldots \approx 14.9$ cm **A1**

Warning: Forgetting to square-root after isolating r^2 is a frequent error; always check units and the reasonableness of the answer.

(b) Arc length $AB = \dfrac{72}{360} \times 2\pi r = \dfrac{72}{360} \times 2\pi \times 14.9070\ldots$ **M1**

Unrounded $= 18.7336\ldots \approx 18.7$ cm **A1**

Warning: Using the rounded $r = 14.9$ rather than the unrounded value propagates a rounding error into the arc length.

(c) For the second sector: arc $= \dfrac{50}{360} \times 2\pi r_2 = 18.7336\ldots \Rightarrow r_2 = \dfrac{18.7336\ldots \times 360}{50 \times 2\pi}$: unrounded $= 21.4664\ldots \approx 21.5$ cm **A1**

Warning: Using 50° in the area formula instead of the arc-length formula confuses the two sector properties.

Q29 — Arc lengths on circular track, sector cost [HARD]

(a) Arc AB: $l_{AB} = \frac{84}{360} \times 2\pi \times 50$: unrounded $= 73.303\ldots \approx 73.3$ m **A1**

(b) Angle $\angle BOC = 210^\circ - 84^\circ = 126^\circ$ (since both are measured from A in the same direction). **M1**

Arc $BC = \frac{126}{360} \times 2\pi \times 50$: unrounded $= 109.956\ldots \approx 110$ m **A1**

Warning: Using the reflex angle $360^\circ - 126^\circ = 234^\circ$ would give the longer arc, not the shorter arc BC as required.

(c) Area of sector $OAB = \frac{84}{360} \times \pi \times 50^2$: unrounded $= 1832.59\ldots$ m^2 **M1**

Cost $= 1832.59\ldots \times 12.50 = \$22907.4\ldots \approx \$22907$ **A1**

Warning: Using the arc length rather than the sector area to compute the painting cost conflates perimeter and area; read the question carefully for the correct measure.

Q30 — Perpendicular bisector and simultaneous equations [HARD]

(a) Midpoint $M = \left(\frac{2+8}{2}, \frac{9+3}{2}\right) = (5, 6)$. **A1**

Gradient of JK: $\frac{3-9}{8-2} = \frac{-6}{6} = -1$. **A1**

(b) Perpendicular gradient $= 1$ (negative reciprocal of -1). Through $M(5,6)$: $y - 6 = 1(x-5) \Rightarrow y = x + 1$. **A1**

Warning: The negative reciprocal of -1 is $+1$, not -1; this sign flip is easy to miss when the gradient is ± 1.

(c) Solve simultaneously: $y = x + 1$ and $y = 2x - 3$. **M1**

$x + 1 = 2x - 3 \Rightarrow x = 4$, $y = 5$. Casualty at $(4, 5)$, i.e. 400 m east and 500 m north of the grid origin. **A1**

Warning: Failing to substitute back to find both coordinates, or giving only $x = 4$, loses the accuracy mark.