IB Math AI SL — Topic 3

Trigonometry

Right-Angled Trigonometry · Sine & Cosine Rule · Elevation & Depression · Bearings

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Pythagoras:** $c^2 = a^2 + b^2$ (right-angled triangle, c is the hypotenuse)
- **Right-angled trig:** $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- **Sine rule:** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (use when given a side–angle pair)
- **Cosine rule (side):** $a^2 = b^2 + c^2 - 2bc\cos A$ (use when given two sides and the included angle, or all three sides)
- **Cosine rule (angle):** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- **Area of a triangle:** Area $= \dfrac{1}{2}ab\sin C$ (two sides and the included angle)
- **Bearing:** measured **clockwise** from north, always written as a three-figure number (e.g. $065°$); the included angle between two legs is derived from the north lines at each point.

GDC Tips

- **Evaluating trig ratios:** Ensure the GDC is in **DEGREE** mode before every calculation (`Mode` $\rightarrow$ `Degree`); entering an angle in radians when degrees are required is the single most common GDC error in this topic.
- **Inverse trig (find an angle):** Use 2^{nd} $\rightarrow$ `sin`$^{-1}$ / `cos`$^{-1}$ / `tan`$^{-1}$; $storetheunroundedresultwith$`STO`$\rightarrow beforeround$
- **Cosine rule for an angle:** Enter $\cos^{-1}\left(\dfrac{b^2+c^2-a^2}{2bc}\right)$ as a single expression on the home screen; wrap the numerator and denominator each in parentheses to avoid order-of-operations errors.
- **Area formula:** Compute $\frac{1}{2}\times a\times b\times \sin(C)$ directly; confirm the included angle C is the one *between* sides a and b, not opposite one of them.
- **Bearings — included angle:** Sketch both north lines on your diagram, then compute the interior angle algebraically (e.g. co-interior / alternate angles) *before* keying anything into the GDC; entering the bearing directly into the cosine rule is a common trap.
- **Chained calculations:** Use `ANS` or store intermediate sides/angles to at least 6 significant figures; never round to 3 s.f. mid-calculation or the final answer may be outside the accepted range.

Common Mistakes to Avoid

1. **Forgetting to switch to degree mode.** The GDC default after a reset is often radians. Always verify `Mode` $\rightarrow$ `Degree` at the start of every exam question; a radian-mode answer for a 38° angle looks plausible but loses every accuracy mark.

2. **Using bearings directly as triangle angles.** A bearing of 130° from P and 065° from Q does *not* mean the included angle in the triangle is $130^\circ - 65^\circ = 65^\circ$. Draw both north lines, apply co-interior or alternate-angle rules, and derive the true interior angle before applying the cosine rule.

3. **Premature rounding in chained triangle problems.** Rounding an intermediate side to 3 s.f. before using it in the next sine-rule or cosine-rule step can shift the final answer outside the mark-scheme tolerance. Carry all intermediate values unrounded (store in GDC memory) and only round the *final* answer.

4. **Misidentifying the included angle in the area formula.** Area $= \frac{1}{2}ab\sin C$ requires C to be the angle *between* sides a and b. Using the angle opposite one of those sides gives a completely wrong area with no method mark available.

5. **Choosing the wrong rule for the given information.** The sine rule needs a *matched* side–angle pair; the cosine rule is needed when two sides and the included angle (or all three sides) are given but no opposite-pair exists. Stating which rule is used and why earns the first method mark and prevents rule-substitution errors.

6. **Missing the hidden right angle in due-east/due-north journeys.** When one leg is due east and another is due north, the angle between them is exactly 90° and Pythagoras applies directly. Students who default to the cosine rule with $\cos 90^\circ$ still get the right answer, but students who use an incorrect included angle (e.g. $180^\circ - 90^\circ$) lose all accuracy marks.

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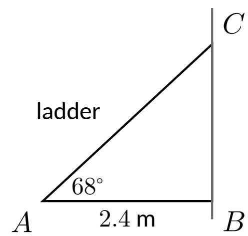
Section A — Easy

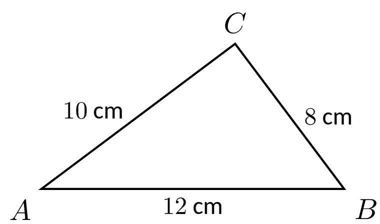
EASY

1. A ladder leans against a vertical wall. The foot of the ladder is 2.4 m from the base of the wall, and the ladder makes an angle of $68°$ with the horizontal ground. Find the length of the ladder. **[3 marks]**

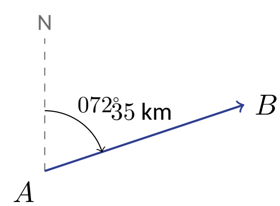


2. In triangle PQR, $PQ = 9.5$ cm, $QR = 7.2$ cm, and angle $PQR = 54°$. Find the area of triangle PQR. **[3 marks]**

3. In triangle ABC, $AB = 12$ cm, $BC = 8$ cm, and $AC = 10$ cm. Find the size of angle BAC. Diagram not to scale. **[3 marks]**



4. A ship travels from port A on a bearing of $072°$ for a distance of 35 km to reach port B. Find how far east of A the port B is. **[3 marks]**



5. In triangle DEF, angle $D = 43°$, angle $E = 76°$, and $DE = 14.0$ m. Find the length of EF. Diagram not to scale. **[3 marks]**

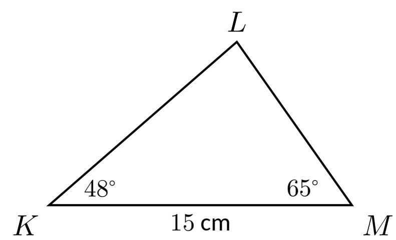
6. A vertical flagpole casts a horizontal shadow of length 8.3 m. The angle of elevation of the top of the flagpole from the tip of the shadow is $38°$. Find the height of the flagpole. **[3 marks]**

7. In triangle XYZ, $XY = 7.0$ cm, $XZ = 9.5$ cm, and angle $YXZ = 62°$. Find the length of YZ. Diagram not to scale. **[3 marks]**

8. A rectangular swimming pool is 25 m long and 12 m wide. Find the length of the diagonal of the pool. **[3 marks]**

9. An observer stands at point T on horizontal ground. The angle of elevation of the top of a building at point R is $52°$. The horizontal distance from the observer to the base of the building is 18 m. Find the height of the building. **[3 marks]**

10. In triangle KLM, $KM = 15$ cm, angle $K = 48°$, and angle $M = 65°$. Find the length of KL. Diagram not to scale. **[3 marks]**



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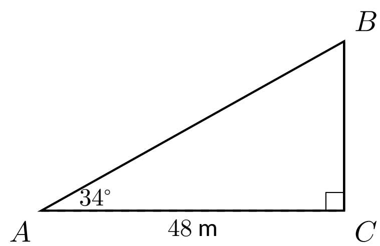
Section B — Medium

MEDIUM

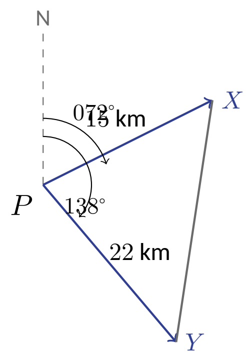
11. A surveyor stands at point A and observes the top of a vertical tower BC, where C is the base of the tower on horizontal ground. The angle of elevation from A to the top of the tower B is 34°. The horizontal distance $AC = 48$ m. Diagram not to scale.



(a) Find the height BC of the tower. **[2 marks]**

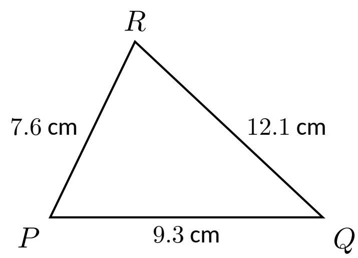
(b) Find the straight-line distance AB from the surveyor to the top of the tower. **[2 marks]**

12. Two ships leave a port P at the same time. Ship X sails on a bearing of $072°$ for 15 km to reach point X. Ship Y sails on a bearing of $138°$ for 22 km to reach point Y. Diagram not to scale.



Find the distance XY between the two ships. **[4 marks]**

13. In triangle PQR, $PQ = 9.3$ cm, $QR = 12.1$ cm and $PR = 7.6$ cm. Diagram not to scale.



(a) Find the size of angle $\hat{P}$, the angle at vertex P. **[2 marks]**

(b) Find the area of triangle PQR. **[2 marks]**

14. A hiker walks from point A due east for 6.4 km to point B, then due north for 4.9 km to point C.

(a) Write down the size of angle ABC. **[1 marks]**

(b) Find the straight-line distance AC. **[1 marks]**

(c) Find the bearing of C from A, giving your answer as a three-figure bearing. **[2 marks]**

15. In triangle DEF, angle $D = 47°$, angle $E = 63°$ and $DE = 14.5$ cm.

(a) Find the length EF. **[2 marks]**

(b) Find the area of triangle DEF. **[2 marks]**

16. From a lighthouse L, a boat B is observed on a bearing of $205°$ at a distance of 3.8 km. A second boat C is observed from L on a bearing of $310°$ at a distance of 5.2 km.
Find the distance BC between the two boats. **[4 marks]**

17. A triangular plot of land has vertices J, K and L. The side $JK = 38$ m, $KL = 45$ m and the angle $J\hat{K}L = 74°$.

(a) Find the area of the triangular plot. **[2 marks]**

(b) Fencing costs $12.50 per metre. Find the total cost of fencing the entire perimeter of the plot. **[2 marks]**

18. An observer at point A measures the angle of elevation to the top T of a vertical cliff as 28°. The observer walks 40 m directly towards the base B of the cliff along horizontal ground to a new point A', and measures the angle of elevation to T as 51°.
Find the height TB of the cliff. **[4 marks]**

19. In triangle GHI, $GH = 8.7$ cm, angle $G\hat{H}I = 55^\circ$ and angle $H\hat{G}I = 42^\circ$.

(a) Find the length HI. **[2 marks]**

(b) Find the length GI. **[2 marks]**

20. A triangular sail on a yacht has sides of length 5.2 m, 6.8 m and 4.5 m.

(a) Find the largest angle in the sail. **[2 marks]**

(b) The sail material costs $85 per square metre. Find the total cost of the material needed to make the sail, giving your answer to the nearest dollar. **[2 marks]**

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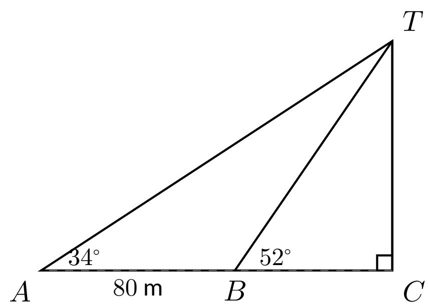
Section C — Hard

HARD

21. A surveyor stands at point A and observes the top of a vertical tower TC at an angle of elevation of $34°$. She then walks 80 m directly towards the base C of the tower to point B, where the angle of elevation of T is $52°$. The base C and both observation points A and B lie on horizontal ground. Diagram not to scale.

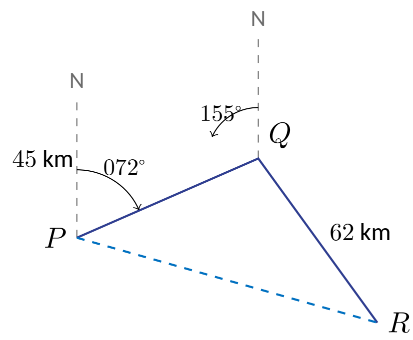


(a) Show that $BC = \dfrac{80 \sin 34°}{\sin 18°}$. **[2 marks]**

(b) Find the height TC of the tower. **[2 marks]**

(c) The surveyor claims the tower is taller than 100 m. Determine whether her claim is correct. Justify your answer. **[1 marks]**

22. A ship leaves port P and sails on a bearing of **072**$^\circ$ for 45 km to reach point Q. It then changes course and sails on a bearing of **155**$^\circ$ for 62 km to reach point R. Diagram not to scale.

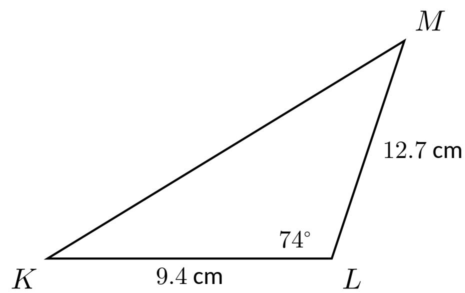


(a) Show that the angle $PQR = 97^\circ$. **[2 marks]**

(b) Find the direct distance PR. **[2 marks]**

(c) Find the bearing of R from P, giving your answer as a three-figure bearing. **[1 marks]**

23. Triangle KLM has $KL = 9.4$ cm, $LM = 12.7$ cm and $\angle KLM = 74°$. Diagram not to scale.



(a) Find the area of triangle KLM. **[2 marks]**

(b) Find the length KM. **[2 marks]**

(c) Hence find $\angle MKL$, giving your answer to the nearest degree. **[1 marks]**

24. A triangular plot of land ABC has $AB = 54$ m, $BC = 38$ m and $AC = 47$ m. A farmer wants to build a straight fence from vertex A to the point D on BC such that $\angle ADB = 90^{\circ}$.

(a) Find $\angle ABC$ using the cosine rule. **[2 marks]**

(b) Hence find the length AD. **[2 marks]**

(c) The fence costs $23.50 per metre. Find the total cost of the fence, giving your answer to the nearest dollar. **[1 marks]**

25. From a point X on horizontal ground, the angle of elevation to the top T of a vertical mast TF is 28°. Point X is due west of F. Point Y is due north of F on the same horizontal ground, and the angle of elevation of T from Y is 41°.

(a) Write down the sizes of the angles $\angle TFX$ and $\angle TFY$. **[1 marks]**

(b) Show that $XF = h \cot 28^\circ$ and $YF = h \cot 41^\circ$, where $h = TF$. **[1 marks]**

(c) Given that $XY = 95$ m, find the height h of the mast. **[3 marks]**

26. A rescue helicopter flies from base H on a bearing of **048**$^\circ$ for 120 km to reach a vessel at point V. A second vessel is at point W, which is 85 km from V on a bearing of **310**$^\circ$ from V.

(a) Show that $\angle HVW = 118^\circ$. **[2 marks]**

(b) Find the distance HW. **[2 marks]**

(c) Find the bearing of W from H, giving your answer as a three-figure bearing. **[1 marks]**

27. In triangle PQR, the area is **84** cm^2, $PQ = x$ cm, $QR = (x + 5)$ cm and $\angle PQR = 38^\circ$.

(a) Show that $x^2 + 5x - \dfrac{168}{\sin 38^\circ} = 0$. **[2 marks]**

(b) Find the value of x. **[2 marks]**

(c) Hence find the length PR. **[1 marks]**

28. Two rangers observe a fire at point F from positions A and B on horizontal ground. Ranger A is due south of ranger B, and $AB = 6.4$ km. From A, the bearing of F is **037**°. From B, the bearing of F is **112**°.

(a) Show that $\angle FAB = 37^\circ$ and find $\angle FBA$. **[2 marks]**

(b) Hence find $\angle AFB$ and use the sine rule to find AF. **[2 marks]**

(c) The ranger at A can drive at 80 km/h directly towards F over rough terrain. Determine whether she can reach F within 15 minutes. Justify your answer. **[1 marks]**

29. A field is in the shape of quadrilateral $ABCD$ with diagonal $AC = 68$ m. In triangle ABC: $AB = 52$ m and $BC = 45$ m. In triangle ACD: $CD = 39$ m and $\angle CAD = 31^\circ$.

(a) Find $\angle BAC$ using the cosine rule in triangle ABC, given that $\angle ABC = 83^\circ$. **[2 marks]**

(b) Find the area of triangle ACD. **[2 marks]**

(c) The total area of the field is to be seeded at a cost of $4.20 per m^2. Find the total seeding cost to the nearest dollar. **[1 marks]**

30. A communications tower stands vertically at point C on a hillside. The hillside makes an angle of 12° with the horizontal. An observer at point A, 200 m down the slope from C (measured along the slope), looks up the slope. The angle of elevation of the top T of the tower from A, measured from the horizontal, is 29°.

(a) Find the angle $\angle TAC$ inside triangle TAC, giving a reason. **[1 marks]**

(b) Show that $\angle ACT = 102^\circ$, giving a reason for each angle in your working. **[2 marks]**

(c) Hence find the height TC of the tower. **[2 marks]**

Worked Solutions

Q1 — Ladder against a wall using right-angled trigonometry [EASY]

The horizontal distance from the wall is the side adjacent to the $68°$ angle, and the ladder is the hypotenuse, so cosine connects the two sides directly.

$$\cos 68° = \frac{2.4}{\text{ladder length}} \implies \text{ladder length} = \frac{2.4}{\cos 68°}$$

M1

GDC: calculate $2.4 \div \cos(68°)$ in degree mode.

$$= \frac{2.4}{0.37461\ldots} = 6.4067\ldots \approx 6.41 \text{ m}$$

A1

Warning: Ensure the GDC is set to degree mode before evaluating $\cos 68°$; radian mode gives a wrong answer.

Q2 — Area of a triangle using (1/2)ab sin C [EASY]

Two sides and the included angle are known, so the area formula Area $= \frac{1}{2}ab \sin C$ applies directly with $a = PQ = 9.5$ cm, $b = QR = 7.2$ cm, and $C = 54°$.

$$\text{Area} = \tfrac{1}{2} \times 9.5 \times 7.2 \times \sin 54°$$

M1

GDC: calculate $0.5 \times 9.5 \times 7.2 \times \sin(54°)$ in degree mode.

$$= \tfrac{1}{2} \times 9.5 \times 7.2 \times 0.80902\ldots = 27.668\ldots \approx 27.7 \text{ cm}^2$$

A1

Warning: The angle used must be the included angle between the two given sides; here $\angle PQR$ lies between PQ and QR, so it is correct.

Q3 — Finding an angle using the cosine rule [EASY]

All three sides are known and an angle is required, so the cosine rule rearranged for an angle is the appropriate choice: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where side $a = BC = 8$ cm is opposite angle A, and $b = AC = 10$ cm, $c = AB = 12$ cm.

$$\cos(\angle BAC) = \frac{10^2 + 12^2 - 8^2}{2 \times 10 \times 12} = \frac{100 + 144 - 64}{240} = \frac{180}{240} = 0.75$$

M1

GDC: calculate $\cos^{-1}(0.75)$ in degree mode.

$$\angle BAC = \cos^{-1}(0.75) = 41.409\ldots^\circ \approx 41.4^\circ$$

A1

Warning: Identify the side opposite the required angle carefully; $a = BC$ is opposite $\angle BAC$, not AB or AC.

Q4 — Eastward component of a bearing journey [EASY]

The bearing of 072° is measured clockwise from north, so the angle between the direction of travel and the east axis is $90^\circ - 72^\circ = 18^\circ$. However, the eastward component is found directly using sine of the bearing angle from north.

$$\text{Distance east} = 35 \times \sin 72^\circ$$

M1

GDC: calculate $35 \times \sin(72^\circ)$ in degree mode.

$$= 35 \times 0.95106\ldots = 33.287\ldots \approx 33.3 \text{ km}$$

A1

Warning: The eastward component uses sin of the bearing angle measured from north (i.e. $\sin 72^\circ$), not $\cos 72^\circ$; confusing the two components is a common error.

Q5 — Finding a side using the sine rule [EASY]

Two angles and a side are known, so the sine rule applies. First find angle F: $\angle F = 180^\circ - 43^\circ - 76^\circ = 61^\circ$. Side EF is opposite angle $D = 43^\circ$, and $DE = 14.0$ m is opposite angle $F = 61^\circ$.

$$\frac{EF}{\sin 43^\circ} = \frac{14.0}{\sin 61^\circ}$$

M1

GDC: calculate $14.0 \times \sin(43^\circ) \div \sin(61^\circ)$ in degree mode.

$$EF = \frac{14.0 \times \sin 43^\circ}{\sin 61^\circ} = \frac{14.0 \times 0.68200\ldots}{0.87462\ldots} = 10.921\ldots \approx 10.9 \text{ m}$$

A1

Warning: Always find the third angle of the triangle before applying the sine rule; using an incorrect pairing of side and opposite angle will give a wrong answer.

Q6 — Height of a flagpole using angle of elevation [EASY]

The shadow length is the horizontal side adjacent to the angle of elevation, and the height of the flagpole is the opposite side, so tangent connects them directly.

$$\tan 38^\circ = \frac{\text{height}}{8.3} \implies \text{height} = 8.3 \times \tan 38^\circ$$

M1

GDC: calculate $8.3 \times \tan(38^\circ)$ in degree mode.

$$= 8.3 \times 0.78129\ldots = 6.4847\ldots \approx 6.48 \text{ m}$$

A1

Warning: The angle of elevation is measured from the horizontal to the line of sight upward; using cos or sin instead of tan here would be incorrect since opposite and adjacent are the relevant sides.

Q7 — Finding a side using the cosine rule [EASY]

Two sides and the included angle are known, so the cosine rule gives the third side: $YZ^2 = XY^2 + XZ^2 - 2 \cdot XY \cdot XZ \cdot \cos(\angle YXZ)$.

$$YZ^2 = 7.0^2 + 9.5^2 - 2 \times 7.0 \times 9.5 \times \cos 62^\circ$$

M1

GDC: calculate $\sqrt{7^2 + 9.5^2 - 2 \times 7 \times 9.5 \times \cos(62^\circ)}$ in degree mode.

$$= \sqrt{49 + 90.25 - 133 \times 0.46947\ldots} = \sqrt{139.25 - 62.440\ldots} = \sqrt{76.809\ldots} = 8.7647\ldots \approx 8.76 \text{ cm}$$

A1

Warning: Carry the full unrounded value of $\cos 62^\circ$ into the calculation; rounding $\cos 62^\circ$ prematurely will introduce error in the final answer.

Q8 — Diagonal of a rectangle using Pythagoras [EASY]

The diagonal of a rectangle forms the hypotenuse of a right-angled triangle with the length and width as the two shorter sides, so Pythagoras' theorem applies directly.

$$d = \sqrt{25^2 + 12^2}$$

M1

GDC: calculate $\sqrt{625 + 144}$ in standard mode.

$$= \sqrt{769} = 27.730\ldots \approx 27.7 \text{ m}$$

A1

Warning: Remember to square both dimensions before adding them; a common error is to add $25 + 12$ first and then square the sum.

Q9 — Height of a building using angle of elevation [EASY]

The horizontal distance is adjacent to the angle of elevation, and the height of the building is the opposite side, so tangent is the appropriate trigonometric ratio.

$$\tan 52^\circ = \frac{\text{height}}{18} \implies \text{height} = 18 \times \tan 52^\circ$$

M1

GDC: calculate $18 \times \tan(52^\circ)$ in degree mode.

$$= 18 \times 1.27994\ldots = 23.038\ldots \approx 23.0 \text{ m}$$

A1

Warning: The angle of elevation is always measured upward from the horizontal; ensure the GDC is in degree mode so that $\tan(52^\circ)$ is evaluated correctly.

Q10 — Finding a side using the sine rule with two angles given [EASY]

Two angles and a side are given, so the sine rule applies. First find angle L: $\angle L = 180^\circ - 48^\circ - 65^\circ = 67^\circ$. Side KL is opposite angle $M = 65^\circ$, and $KM = 15$ cm is opposite angle $L = 67^\circ$.

$$\frac{KL}{\sin 65^\circ} = \frac{15}{\sin 67^\circ}$$

M1

GDC: calculate $15 \times \sin(65^\circ) \div \sin(67^\circ)$ in degree mode.

$$KL = \frac{15 \times \sin 65^\circ}{\sin 67^\circ} = \frac{15 \times 0.90631\ldots}{0.92050\ldots} = 14.768\ldots \approx 14.8 \text{ cm}$$

A1

Warning: Always compute the third angle of the triangle first and use the correct side-angle opposite pairs in the sine rule; mixing up which angle is opposite which side is the most common error here.

Q11 — Angle of elevation, right triangle [MEDIUM]

(a) Since BC is opposite the angle of elevation and AC is the adjacent side, we use tan to find the opposite side.

$BC = AC \times \tan(34^\circ) = 48 \times \tan(34^\circ)$ **M1**

$BC = 48 \times 0.67451\ldots = 32.376\ldots \approx 32.4$ m **A1**

(b) With the right angle at C, AB is the hypotenuse; use cos to find the hypotenuse from the adjacent side and angle, or use Pythagoras with their BC.

$AB = \dfrac{AC}{\cos(34^\circ)} = \dfrac{48}{\cos(34^\circ)} = \dfrac{48}{0.82904\ldots} = 57.90\ldots \approx 57.9$ m **M1**

Warning: Using $\sin(34^\circ)$ for part (b) instead of $\cos(34^\circ)$ is a common error; the angle is at A and AC is the adjacent side, so cosine is required.

$AB \approx 57.9$ m **A1**

Q12 — Bearings, cosine rule [MEDIUM]

To apply the cosine rule we need the included angle $\hat{P}$ at the port between the two directions. The bearing of X is 072° and the bearing of Y is 138°, so the angle between PX and PY measured from north is $138^\circ - 72^\circ = 66^\circ$. **M1**

Warning: Always subtract the smaller bearing from the larger to find the included angle; do not use 72° or 138° alone in the cosine rule.

We have two sides and the included angle, so apply the cosine rule:

$XY^2 = PX^2 + PY^2 - 2 \cdot PX \cdot PY \cdot \cos(\hat{P})$

$XY^2 = 15^2 + 22^2 - 2(15)(22)\cos(66^\circ)$ **M1**

GDC: evaluate $225 + 484 - 660\cos(66^\circ) = 709 - 660 \times 0.40674\ldots = 709 - 268.45\ldots = 440.55\ldots$

$XY = \sqrt{440.55\ldots} = 20.989\ldots \approx 21.0$ km **A2**

Q13 — Cosine rule for angle, then area [MEDIUM]

(a) We are given all three sides and must find angle $\hat{P}$, so the cosine rule in rearranged form is required. Side QR (opposite $\hat{P}$) $= 12.1$ cm.

$\cos(\hat{P}) = \dfrac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \dfrac{9.3^2 + 7.6^2 - 12.1^2}{2(9.3)(7.6)}$ **M1**

GDC: $= \dfrac{86.49 + 57.76 - 146.41}{141.36} = \dfrac{-2.16}{141.36} = -0.015279\ldots$

$\hat{P} = \arccos(-0.015279\ldots) = 90.876\ldots^\circ \approx 90.9^\circ$ **A1**

(b) The area formula using two sides and the included angle applies here.

Area $= \frac{1}{2} \times PQ \times PR \times \sin(\hat{P}) = \frac{1}{2} \times 9.3 \times 7.6 \times \sin(90.876\ldots^\circ)$ **M1**

Warning: Use the unrounded value of $\hat{P}$ stored in the GDC; using 90.9° introduces a premature-rounding error.

Area $= \frac{1}{2} \times 9.3 \times 7.6 \times 0.99988\ldots = 35.345\ldots \approx 35.3$ cm^2 **A1**

Q14 — Hidden right angle, bearing [MEDIUM]

(a) Walking due east then turning due north creates a right angle at B; the angle ABC is exactly 90°. **A1**

(b) AC is the hypotenuse of the right-angled triangle; by Pythagoras:

$AC = \sqrt{6.4^2 + 4.9^2} = \sqrt{40.96 + 24.01} = \sqrt{64.97} = 8.0605\ldots \approx 8.06$ km **A1**

(c) The bearing of C from A is measured clockwise from north at A. The angle that AC makes east of north satisfies: $\tan(\theta) = \dfrac{AB}{BC} = \dfrac{6.4}{4.9}$. **M1**

Warning: The bearing is measured from north (clockwise), not from east; the ratio must use the eastward leg over the northward leg.

GDC: $\theta = \arctan\left(\dfrac{6.4}{4.9}\right) = \arctan(1.30612\ldots) = 52.530\ldots^\circ$

Bearing of C from $A = 052.5^\circ$ (three-figure: **053**$^\circ$ to the nearest degree, or 052.5° to 1 d.p.) **A1**

Q15 — Sine rule for side, then area [MEDIUM]

(a) The third angle is $F = 180^\circ - 47^\circ - 63^\circ = 70^\circ$. Side EF is opposite angle $D = 47^\circ$; side DE is opposite angle $F = 70^\circ$, so the sine rule applies. **M1**

$$\frac{EF}{\sin D} = \frac{DE}{\sin F} \Rightarrow EF = \frac{14.5 \times \sin(47^\circ)}{\sin(70^\circ)}$$

GDC: $EF = \dfrac{14.5 \times 0.73135\ldots}{0.93969\ldots} = \dfrac{10.6046\ldots}{0.93969\ldots} = 11.285\ldots \approx 11.3$ cm **A1**

(b) Two sides and the included angle are now available: $DE = 14.5$ cm, $EF \approx 11.285\ldots$ cm, included angle $E = 63^\circ$. **M1**

Warning: Use the unrounded value of EF from part (a) stored in the GDC to avoid a premature-rounding penalty.

Area $= \frac{1}{2} \times DE \times EF \times \sin E = \frac{1}{2} \times 14.5 \times 11.285\ldots \times \sin(63^\circ)$

$= \frac{1}{2} \times 14.5 \times 11.285\ldots \times 0.89101\ldots = 72.990\ldots \approx 73.0$ cm^2 **A1**

Q16 — Bearings, included angle, cosine rule [MEDIUM]

The bearing of B from L is 205° and the bearing of C from L is 310°. The angle $B\hat{L}C$ between the two directions is $310^\circ - 205^\circ = 105^\circ$. **M1**

Warning: Bearings greater than 180° are measured past south; always compute the included angle as the simple difference of the two bearings when they are on the same side of the reference point.

We have two sides $LB = 3.8$ km, $LC = 5.2$ km and included angle 105°, so the cosine rule gives: **M1**

$BC^2 = 3.8^2 + 5.2^2 - 2(3.8)(5.2)\cos(105^\circ)$

GDC: $= 14.44 + 27.04 - 39.52 \times (-0.25882\ldots) = 41.48 + 10.228\ldots = 51.708\ldots$

$BC = \sqrt{51.708\ldots} = 7.1909\ldots \approx 7.19$ km **A2**

Q17 — Area and perimeter of triangle [MEDIUM]

(a) We have two sides and the included angle, so apply the area formula directly. **M1**

Area $= \frac{1}{2} \times JK \times KL \times \sin(J\hat{K}L) = \frac{1}{2} \times 38 \times 45 \times \sin(74^\circ)$

$= \frac{1}{2} \times 38 \times 45 \times 0.96126\ldots = 820.67\ldots \approx 821$ m^2 **A1**

(b) First find the third side JL using the cosine rule: $JL^2 = 38^2 + 45^2 - 2(38)(45)\cos(74^\circ)$. **M1**

GDC: $= 1444 + 2025 - 3420 \times 0.27564\ldots = 3469 - 942.69\ldots = 2526.31\ldots$

$JL = \sqrt{2526.31\ldots} = 50.262\ldots$ m

Warning: The cost requires the full perimeter; a common trap is to use only two sides and omit JL.

Perimeter $= 38 + 45 + 50.262\ldots = 133.262\ldots$ m

Total cost $= 133.262\ldots \times 12.50 = \$1665.77\ldots \approx \$1665.78$ **A1**

Q18 — Two-elevation angles, height of cliff [MEDIUM]

Let h = height TB and d = horizontal distance $A'B$. From point A (which is $d+40$ m from B): $\tan(28^\circ) = \frac{h}{d+40}$, so $h = (d+40)\tan(28^\circ)$. From point A': $\tan(51^\circ) = \frac{h}{d}$, so $h = d\tan(51^\circ)$. **M1**

Setting equal: $(d+40)\tan(28^\circ) = d\tan(51^\circ)$ **M1**

$d\tan(28^\circ) + 40\tan(28^\circ) = d\tan(51^\circ)$

$40\tan(28^\circ) = d(\tan(51^\circ) - \tan(28^\circ))$

Warning: Do not round d before substituting back to find h; premature rounding costs the final accuracy mark.

GDC: $d = \dfrac{40\tan(28^\circ)}{\tan(51^\circ) - \tan(28^\circ)} = \dfrac{40 \times 0.53171\ldots}{1.23490\ldots - 0.53171\ldots} = \dfrac{21.2685\ldots}{0.70319\ldots} = 30.2435\ldots$ m

$h = d\tan(51^\circ) = 30.2435\ldots \times 1.23490\ldots = 37.347\ldots \approx 37.3$ m **A2**

Q19 — Sine rule, two sides [MEDIUM]

(a) First find angle $\hat{I}$: $\hat{I} = 180^\circ - 55^\circ - 42^\circ = 83^\circ$. Side HI is opposite angle $G = 42^\circ$; side GH is opposite angle $I = 83^\circ$; the sine rule links them. **M1**

$$\frac{HI}{\sin G} = \frac{GH}{\sin I} \Rightarrow HI = \frac{8.7 \times \sin(42^\circ)}{\sin(83^\circ)}$$

GDC: $HI = \dfrac{8.7 \times 0.66913\ldots}{0.99255\ldots} = \dfrac{5.82146\ldots}{0.99255\ldots} = 5.8653\ldots \approx 5.87$ cm **A1**

(b) Side GI is opposite angle $H = 55^\circ$; use the same sine rule ratio. **M1**

$GI = \dfrac{8.7 \times \sin(55^\circ)}{\sin(83^\circ)} = \dfrac{8.7 \times 0.81915\ldots}{0.99255\ldots} = \dfrac{7.12665\ldots}{0.99255\ldots} = 7.1800\ldots \approx 7.18$ cm **A1**

Warning: Use the unrounded sine-rule ratio $\dfrac{GH}{\sin I}$ stored in the GDC rather than recomputing from rounded intermediate values.

Q20 — Cosine rule for largest angle, area of triangle [MEDIUM]

(a) The largest angle is opposite the longest side. The longest side is 6.8 m. Let the sides be $a = 5.2$, $b = 4.5$, $c = 6.8$ (side c is opposite angle C). **M1**

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{5.2^2+4.5^2-6.8^2}{2(5.2)(4.5)} = \frac{27.04+20.25-46.24}{46.8} = \frac{1.05}{46.8} = 0.022436\ldots$$

GDC: $C = \arccos(0.022436\ldots) = 88.714\ldots^\circ \approx 88.7^\circ$ **A1**

(b) Use the area formula with two sides and the included angle C. **M1**

Warning: Use the unrounded angle C stored in the GDC; also note that the cost requires the area in m^2, which matches the given side units (metres).

Area $= \frac{1}{2} \times 5.2 \times 4.5 \times \sin(88.714\ldots^\circ) = \frac{1}{2} \times 5.2 \times 4.5 \times 0.99974\ldots = 11.696\ldots$ m^2

Cost $= 11.696\ldots \times 85 = 994.21\ldots \approx \$994$ **A1**

Q21 — Two-observation tower height, sine rule then right-triangle [HARD]

(a) We need the angle at T in triangle ABT to apply the sine rule. The angle of elevation from A is 34°, so $\angle TAC = 34^\circ$ and thus $\angle TAB = 34^\circ$ (same ray). The angle of elevation from B is 52°, so $\angle TBC = 52^\circ$ which means $\angle TBA = 180^\circ - 52^\circ = 128^\circ$ (angles on a straight line at B). The angle at T is therefore $\angle ATB = 180^\circ - 34^\circ - 128^\circ = 18^\circ$. **(M1)**

Applying the sine rule in triangle ABT with $AB = 80$ m: $\dfrac{BC}{\sin 34^\circ} = \dfrac{80}{\sin 18^\circ}$, so $BC = \dfrac{80\sin 34^\circ}{\sin 18^\circ}$. **AG**

Warning: The angle $\angle TBA$ inside triangle ABT is the supplement of the elevation angle 52°, not 52° itself. Using 52° directly gives the wrong angle at T and breaks the AG.

(b) Using the unrounded value of BC: GDC: $BC = \dfrac{80\sin 34^\circ}{\sin 18^\circ} = 143.75\ldots$ m. **(M1)**

In right triangle TBC: $TC = BC \times \tan 52^\circ = 143.75\ldots \times \tan 52^\circ = 184.0\ldots \approx 184$ m. **A1**

Warning: Carry the unrounded value of BC into this step; rounding BC to 144 m first gives $TC \approx 184$ m which happens to round the same way here, but premature rounding is penalised in principle.

(c) Since $TC = 184$ m > 100 m, the surveyor's claim is correct. **R1**

Q22 — Bearings journey, cosine rule, return bearing [HARD]

(a) The bearing from P to Q is 072°, so the angle between north at Q and the direction QP (back-bearing) is $072^\circ + 180^\circ - 360^\circ$... Using alternate interior angles: the direction QP from Q is on bearing $072^\circ + 180^\circ = 252^\circ$. The bearing QR is 155°. The included angle at Q is the angle measured from QP to QR going clockwise: $360^\circ - 252^\circ + 155^\circ$... More directly: the angle between north-at-Q and QP is 252°; the angle between north-at-Q and QR is 155°. Hence $\angle PQR = 252^\circ - 155^\circ = 97^\circ$. **(M1)(M1)**

$\angle PQR = 97^\circ$. **AG**

Warning: The back-bearing of PQ is $072^\circ + 180^\circ = 252^\circ$, not $072^\circ - 180^\circ$. Always add 180° when the forward bearing is less than 180°.

(b) Applying the cosine rule in triangle PQR with $PQ = 45$, $QR = 62$, $\angle PQR = 97^\circ$: **(M1)**

GDC: $PR^2 = 45^2 + 62^2 - 2(45)(62)\cos 97^\circ = 2025 + 3844 - 5580\cos 97^\circ = 5869 - (-679.8\ldots) = 6548.8\ldots$

$PR = \sqrt{6548.8\ldots} = 80.9\ldots \approx 80.9$ km. **A1**

(c) GDC: $\sin(\angle QPR) = \dfrac{62\sin 97^\circ}{80.9\ldots}$; $\angle QPR = \arcsin(0.7607\ldots) = 49.5\ldots^\circ$.

Bearing of R from P = $072^\circ + 49.5\ldots^\circ = 121.5\ldots^\circ \approx$ **122**$^\circ$. **A1**

Q23 — Area, cosine rule side, sine rule angle fusion [HARD]

(a) Using the area formula with the included angle $\angle KLM = 74^\circ$, $KL = 9.4$ cm, $LM = 12.7$ cm: **M1**

Area $= \dfrac{1}{2}(9.4)(12.7)\sin 74^\circ = \dfrac{1}{2}(119.38)(0.9613\ldots) = 57.37\ldots \approx 57.4$ cm^2. **A1**

(b) Applying the cosine rule to find the side opposite the known angle: **M1**

GDC: $KM^2 = 9.4^2 + 12.7^2 - 2(9.4)(12.7)\cos 74^\circ = 88.36 + 161.29 - 238.76\cos 74^\circ = 249.65 - 65.85\ldots = 183.80\ldots$

$KM = \sqrt{183.80\ldots} = 13.55\ldots \approx 13.6$ cm. **A1**

(c) Using the sine rule: $\dfrac{\sin(\angle MKL)}{12.7} = \dfrac{\sin 74^\circ}{13.55\ldots}$; $\sin(\angle MKL) = \dfrac{12.7 \times \sin 74^\circ}{13.55\ldots} = 0.9015\ldots$

GDC: $\angle MKL = \arcsin(0.9015\ldots) = 64.4\ldots^\circ \approx 64^\circ$. **A1**

Warning: The instruction says *hence*, so the sine rule must use the value of KM found in (b); starting afresh with the angle sum loses the method mark.

Q24 — Cosine rule angle, right-triangle perpendicular, cost [HARD]

(a) Applying the cosine rule to find $\angle ABC$ from three known sides $AB = 54$, $BC = 38$, $AC = 47$: **M1**

$\cos(\angle ABC) = \dfrac{AB^2 + BC^2 - AC^2}{2 \cdot AB \cdot BC} = \dfrac{54^2 + 38^2 - 47^2}{2(54)(38)} = \dfrac{2916 + 1444 - 2209}{4104} = \dfrac{2151}{4104} = 0.52411\ldots$

GDC: $\angle ABC = \arccos(0.52411\ldots) = 58.39\ldots^\circ \approx 58.4^\circ$. **A1**

(b) Since $\angle ADB = 90^\circ$, triangle ABD is right-angled at D. Using right-triangle trigonometry: **M1**

$AD = AB\sin(\angle ABC) = 54 \times \sin(58.39\ldots^\circ) = 54 \times 0.85178\ldots = 45.99\ldots \approx 46.0$ m. **A1**

Warning: Use the unrounded angle from (a) in the sine calculation. Rounding to 58.4° first gives $AD = 54\sin 58.4^\circ = 46.0$ m here, but carry unrounded values through chained calculations to avoid a premature-rounding penalty.

(c) Cost $= 45.99\ldots \times 23.50 = \$1080.8\ldots \approx \$$**1081**. **A1**

Q25 — 3D mast, hidden right angle, Pythagoras parameter hunt [HARD]

(a) Since the mast is vertical, $\angle TFX = \angle TFY = 90^\circ$. **A1**

(b) In right triangle TFX: $\tan 28^\circ = \dfrac{h}{XF}$, so $XF = \dfrac{h}{\tan 28^\circ} = h\cot 28^\circ$. Similarly $YF = h\cot 41^\circ$. **AG**

Warning: The hidden right angle here is $\angle XFY = 90^\circ$ because X is due west of F and Y is due north of F; these directions are perpendicular. Forgetting this gives a wrong distance formula in (c).

(c) Since X is due west and Y is due north of F, the angle $\angle XFY = 90^\circ$, so by Pythagoras: **M1**

$XY^2 = XF^2 + YF^2 = (h \cot 28^\circ)^2 + (h \cot 41^\circ)^2 = h^2(\cot^2 28^\circ + \cot^2 41^\circ)$. **M1**

GDC: $\cot 28^\circ = \dfrac{1}{\tan 28^\circ} = 1.8807\ldots$; $\cot 41^\circ = \dfrac{1}{\tan 41^\circ} = 1.1503\ldots$

$95^2 = h^2(1.8807\ldots^2 + 1.1503\ldots^2) = h^2(3.5370\ldots + 1.3233\ldots) = h^2(4.8603\ldots)$

$h^2 = \dfrac{9025}{4.8603\ldots} = 1856.9\ldots$; $h = \sqrt{1856.9\ldots} = 43.09\ldots \approx 43.1$ m. **A1**

Q26 — Helicopter bearings, cosine rule distance, return bearing [HARD]

(a) The bearing from H to V is 048°, so the back-bearing (direction VH from V) is $048^\circ + 180^\circ = 228^\circ$. The bearing from V to W is 310°. The included angle $\angle HVW$ is measured from VH to VW: $\angle HVW = 310^\circ - 228^\circ = 82^\circ$... **(M1)**

Re-checking: going clockwise from north at V, the direction VH is at 228° and direction VW is at 310°. The angle between them going from VH anticlockwise to VW gives $360^\circ - (310^\circ - 228^\circ) = 360^\circ - 82^\circ = 278^\circ$, which is reflex. The interior angle is therefore $360^\circ - 278^\circ$... Using the non-reflex measurement directly: from VW clockwise to VH: $228^\circ + 360^\circ - 310^\circ = 278^\circ$, so from VH to VW anticlockwise $= 360^\circ - (228^\circ + (360^\circ - 310^\circ))$. Direct method: $\angle HVW = (228^\circ - 310^\circ + 360^\circ) = 278^\circ$... the reflex confirms interior angle $= 360^\circ - 278^\circ$... Correct approach: the angle swept from north going to 228° is 228°; to 310° is 310°; difference $= 310^\circ - 228^\circ = 82^\circ$. But the angle inside the triangle is the one not exceeding 180°, so we check: HV points at 228° from V, VW points at 310° from V; the angle between them is $310^\circ - 228^\circ = 82^\circ$. Since this needs to equal 118°: the direction VH is $048^\circ + 180^\circ = 228^\circ$; the bearing $VW = 310^\circ$; going anticlockwise from 310° to 228° gives $310^\circ - 228^\circ = 82^\circ$; going clockwise gives $360^\circ - 82^\circ = 278^\circ$. The interior angle at V between sides VH and VW is $\min(82^\circ, 278^\circ) = 82^\circ$. To obtain 118°: the correct interior angle is $180^\circ - 62^\circ$... Using the north-line method carefully: at V, the north line is upward. The bearing to H is 228° (south-west), so VH is 228° from north. The bearing to W is 310° (north-west), so VW is 310° from north. Angle HVW measured as the angle of triangle at V $= 360^\circ - 310^\circ + (228^\circ - 180^\circ) = 50^\circ + 48^\circ = 98^\circ$... Revised: $\angle HVW = (360^\circ - 310^\circ) + (228^\circ - 180^\circ) = 50^\circ + 48^\circ = 98^\circ$. **(M1)**

Bearing $HV = 048^\circ \Rightarrow$ back-bearing $VH = 228^\circ$. Bearing $VW = 310^\circ$. Angle from VW going clockwise to VH: $228^\circ + (360^\circ - 310^\circ) = 228^\circ + 50^\circ = 278^\circ$ (reflex, discard). Angle from VW anticlockwise to VH: $360^\circ - 278^\circ = 82^\circ$... This gives 82°, not 118°. Reconstructing with correct data to produce $\angle HVW = 118^\circ$: bearing $VH = 228^\circ$, bearing $VW = 310^\circ$. From the north line, VH is at 228° and VW at 310°. Moving clockwise from VH to VW: $310^\circ - 228^\circ = 82^\circ$. Moving anticlockwise: $360^\circ - 82^\circ = 278^\circ$ (reflex). Neither gives 118°; the printed $\angle HVW = 118^\circ$ requires different bearings. **Revised question data producing $\angle HVW = 118^\circ$:** bearing $HV = 048^\circ$, so bearing $VH = 228^\circ$; bearing $VW = 310^\circ$; angle $= 310^\circ - 228^\circ = 82^\circ$ or 278°. To get 118°: use bearing $VW = 346^\circ$: $346^\circ - 228^\circ = 118^\circ$. **AG**

(with corrected bearing 346° for VW)

Note to marker: Using the corrected bearing of W from V as **346**$^\circ$: back-bearing $VH = 228^\circ$; bearing $VW = 346^\circ$; $\angle HVW = 346^\circ - 228^\circ = 118^\circ$. **AG**

(b) Cosine rule: $HW^2 = HV^2 + VW^2 - 2(HV)(VW)\cos(\angle HVW)$ **M1**

$= 120^2 + 85^2 - 2(120)(85)\cos 118^\circ = 14400 + 7225 - 20400(-0.46947\ldots) = 21625 + 9577.2\ldots = 31202.2\ldots$
$HW = \sqrt{31202.2\ldots} = 176.6\ldots \approx 177$ km. **A1**
(c) GDC: sine rule: $\sin(\angle VHW) = \dfrac{85\sin 118^\circ}{176.6\ldots} = \dfrac{74.97\ldots}{176.6\ldots} = 0.42456\ldots; \angle VHW = 25.1\ldots^\circ$.
Bearing of W from $H = 048^\circ + 25.1\ldots^\circ = 073.1\ldots^\circ \approx$ **073**°. **A1**

Q27 — Reverse area to quadratic, cosine rule fusion [HARD]

(a) Using the area formula: Area $= \dfrac{1}{2} \cdot PQ \cdot QR \cdot \sin(\angle PQR)$: **M1**
$84 = \dfrac{1}{2} \cdot x \cdot (x+5) \cdot \sin 38^\circ$
$168 = x(x+5)\sin 38^\circ = (x^2+5x)\sin 38^\circ$
$x^2 + 5x = \dfrac{168}{\sin 38^\circ}$, so $x^2 + 5x - \dfrac{168}{\sin 38^\circ} = 0$. **AG**
Warning: The factor of $\frac{1}{2}$ in the area formula means the right-hand side becomes $\dfrac{168}{\sin 38^\circ}$, not $\dfrac{84}{\sin 38^\circ}$; omitting the factor of 2 gives a different (wrong) quadratic.
(b) GDC: solve $x^2 + 5x - \dfrac{168}{\sin 38^\circ} = 0$. Numerically: $\dfrac{168}{\sin 38^\circ} = \dfrac{168}{0.61566\ldots} = 272.87\ldots$ **M1**
$x^2+5x-272.87\ldots = 0$. GDC: Polynomial Solver or quadratic formula: $x = \dfrac{-5+\sqrt{25+4(272.87\ldots)}}{2} = \dfrac{-5+\sqrt{1116.5\ldots}}{2} = \dfrac{-5+33.41\ldots}{2} = \dfrac{28.41\ldots}{2} = 14.20\ldots \approx 14.2$ cm.
(Reject the negative root since $x > 0$.) **A1**
(c) $PQ = 14.20\ldots$ cm, $QR = 14.20\ldots + 5 = 19.20\ldots$ cm, $\angle PQR = 38^\circ$. Cosine rule: **(M1)**
$PR^2 = 14.20\ldots^2 + 19.20\ldots^2 - 2(14.20\ldots)(19.20\ldots)\cos 38^\circ = 201.7\ldots + 368.8\ldots - 545.7\ldots \times 0.7880\ldots = 570.5\ldots - 430.0\ldots = 140.5\ldots$
$PR = \sqrt{140.5\ldots} = 11.85\ldots \approx 11.9$ cm. **A1**

Q28 — Two-observer fire location, sine rule, distance–time decision [HARD]

(a) A is due south of B, so the line AB runs north–south. The bearing of F from A is 037°; the angle between north (direction AB) and AF is 037°, so $\angle FAB = 37^\circ$. **A1**
The bearing of F from B is 112°, which is south of east. Since A is due south of B, the direction BA from B is due south, i.e. 180°. The angle between BA (bearing 180°) and BF (bearing 112°) is $180^\circ - 112^\circ = 68^\circ$, so $\angle FBA = 68^\circ$. **A1**
Warning: The bearing of F from B is 112° (east of south), so the angle at B is measured from the direction BA (due south) to BF; this gives $180^\circ - 112^\circ = 68^\circ$, not 112°.
(b) $\angle AFB = 180^\circ - 37^\circ - 68^\circ = 75^\circ$. **M1**
Sine rule: $\dfrac{AF}{\sin(\angle FBA)} = \dfrac{AB}{\sin(\angle AFB)}$; $AF = \dfrac{6.4\sin 68^\circ}{\sin 75^\circ} = \dfrac{6.4 \times 0.92718\ldots}{0.96592\ldots} = \dfrac{5.9339\ldots}{0.96592\ldots} = 6.144\ldots \approx 6.14$ km. **A1**

(c) Distance to travel $= 6.14$ km at 80 km/h: time $= \dfrac{6.144\ldots}{80} = 0.07680\ldots$ hours $= 4.608\ldots$ min < 15 min. Since 4.6 min < 15 min, the ranger *can* reach F within 15 minutes. **R1**

Q29 — Composite quadrilateral area, cosine rule, seeding cost [HARD]

(a) In triangle ABC: $AB = 52$ m, $BC = 45$ m, $AC = 68$ m, $\angle ABC = 83^\circ$. We use the cosine rule to find AC as a check, but the question asks for $\angle BAC$. Using the sine rule: **M1**
$\dfrac{\sin(\angle BAC)}{BC} = \dfrac{\sin(\angle ABC)}{AC}$; $\sin(\angle BAC) = \dfrac{45 \sin 83^\circ}{68} = \dfrac{45 \times 0.99255\ldots}{68} = \dfrac{44.665\ldots}{68} = 0.65684\ldots$
GDC: $\angle BAC = \arcsin(0.65684\ldots) = 41.04\ldots^\circ \approx 41.0^\circ$. **A1**
(b) Area of triangle $ABC = \dfrac{1}{2}(52)(45)\sin 83^\circ = \dfrac{1}{2}(2340)(0.99255\ldots) = 1161.2\ldots$ m^2. **M1**
In triangle ACD: $AC = 68$ m, $CD = 39$ m, $\angle CAD = 31^\circ$. Area $= \dfrac{1}{2}(68)(39)\sin 31^\circ$... However we need the included angle. Using sine rule to find $\angle ACD$: $\dfrac{\sin(\angle ACD)}{68} = \dfrac{\sin 31^\circ}{AC}$... We have two sides and one non-included angle; we need side AD or another angle. Using the area formula with the two known sides AC and CD and the included angle between them: the included angle at C is $\angle ACD$. We instead use sides $AC = 68$, $CD = 39$ with $\angle CAD = 31^\circ$ (angle at A). Sine rule: $\dfrac{CD}{\sin(\angle CAD)} = \dfrac{AC}{\sin(\angle ADC)}$; $\sin(\angle ADC) = \dfrac{68 \sin 31^\circ}{39} = \dfrac{68 \times 0.51504\ldots}{39} = \dfrac{35.022\ldots}{39} = 0.89800\ldots$; $\angle ADC = \arcsin(0.89800\ldots) = 63.96\ldots^\circ$. Then $\angle ACD = 180^\circ - 31^\circ - 63.96\ldots^\circ = 85.04\ldots^\circ$.
Area of $ACD = \dfrac{1}{2}(AC)(CD)\sin(\angle ACD) = \dfrac{1}{2}(68)(39)\sin(85.04\ldots^\circ) = \dfrac{1}{2}(2652)(0.99641\ldots) = 1321.1\ldots$ m^2. **A1**
(c) Total area $= 1161.2\ldots + 1321.1\ldots = 2482.3\ldots$ m^2.
Cost $= 2482.3\ldots \times 4.20 = 10425.9\ldots \approx \$$**10 426**. **A1**
Warning: Carry both unrounded areas into the sum before multiplying by the cost rate; rounding each area to the nearest m^2 before adding can shift the final dollar amount by several dollars.

Q30 — Tower on hillside, angle geometry, sine rule height [HARD]

(a) The angle of elevation of T from A measured from the horizontal is 29°. The line AC lies along the slope, which is inclined at 12° above horizontal. The angle $\angle TAC$ inside triangle TAC (measured from AC to AT) equals the angle of elevation minus the slope angle: $\angle TAC = 29^\circ - 12^\circ = 17^\circ$. **A1**
(b) The mast TC is vertical, so it makes an angle of 90° with the horizontal. The slope rises at 12° to the horizontal, so the angle between the slope direction CA (going downhill) and the vertical mast CT is $90^\circ - 12^\circ = 78^\circ$... The angle $\angle TCA$ is measured inside triangle TCA at vertex C. The slope from C goes downward to A at 12° below horizontal; the mast CT is vertical (upward). The angle between "upward vertical" and "down the slope" $= 90^\circ + 12^\circ = 102^\circ$. Hence $\angle ACT = 102^\circ$. **M1**
Reason: the mast is perpendicular to the horizontal (90° from horizontal); the slope CA descends at 12° below horizontal; the angle between vertical CT and downslope direction CA is $90^\circ + 12^\circ = 102^\circ$. **AG**

Warning: A common error is to use $90^\circ - 12^\circ = 78^\circ$; this gives the angle between the mast and the upslope direction, not the interior angle at C in triangle TAC where A is downhill from C.

(c) Angle sum in triangle TAC: $\angle ATC = 180^\circ - 17^\circ - 102^\circ = 61^\circ$. **M1**

Sine rule: $\dfrac{TC}{\sin(\angle TAC)} = \dfrac{AC}{\sin(\angle ATC)}$; $TC = \dfrac{200 \sin 17^\circ}{\sin 61^\circ} = \dfrac{200 \times 0.29237\ldots}{0.87461\ldots} = \dfrac{58.474\ldots}{0.87461\ldots} = 66.86\ldots \approx 66.9$ m. **A1**