IB Math AI SL — Topic 4

Normal Distribution

The Normal Curve · Normal Calculations & Inverse Normal

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________________ Date:________________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

Key Formulae

- **Normal distribution notation:** $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance (note: σ is the standard deviation).
- **Standardisation (z-score):** $z = \dfrac{x-\mu}{\sigma}$, so that $Z \sim N(0,1)$.
- **Symmetry property:** $P(X < \mu) = P(X > \mu) = 0.5$ for any normal distribution.
- **Complement rule:** $P(X > a) = 1 - P(X < a)$.
- **Interval probability:** $P(a < X < b) = P(X < b) - P(X < a)$.
- **Symmetry about the mean:** $P(\mu - k < X < \mu + k) = 2P(\mu < X < \mu + k)$ for any $k > 0$.
- **Expected count from a sample of n:** Expected count $= n \times P(X \in \text{region})$.

GDC Tips

- **Computing $P(a < X < b)$:** Menu $\to$ Statistics $\to$ Distributions $\to$ Normal CDF; enter lower $= a$, upper $= b$, μ, σ. For $P(X < a)$ use lower $= -10^{99}$; for $P(X > a)$ use upper $= 10^{99}$.
- **Inverse normal — finding a threshold x from a given probability:** Menu $\to$ Statistics $\to$ Distributions $\to$ Inverse Normal; enter the cumulative area to the LEFT, μ, σ. Always confirm which tail the question specifies before entering the area.
- **Tail direction for inverse normal:** If the question asks for the top $q\%$ (e.g. "fastest 5%"), the cumulative LEFT area to enter is $1 - 0.01q$, not $0.01q$.
- **Finding an unknown μ or σ from one probability condition:** Use Menu $\to$ Calculator $\to$ Numerical Solve (or nSolve); set up normalcdf$(a, b, \mu, \sigma) = p$ and solve for the unknown parameter directly — algebraic standardisation is not required.
- **Checking symmetry on the GDC:** Verify $P(X < \mu) = 0.5$ by running Normal CDF with lower $= -10^{99}$, upper $= \mu$; the result should display 0.5 exactly, confirming correct entry of μ and σ.

Common Mistakes to Avoid

1. **Wrong tail in inverse normal.** Students enter the given percentage directly as the LEFT-tail area even when the question describes the upper tail (e.g. "top 8%" or "fastest times"). Always ask: is the shaded region on the LEFT or RIGHT? For an upper-tail probability p, enter $1 - p$ into Inverse Normal.
2. **Confusing σ^2 and σ when entering the GDC.** The notation $X \sim N(50, 16)$ means $\mu = 50$ and $\sigma^2 = 16$, so $\sigma = 4$. Entering $\sigma = 16$ into normalcdf or Inverse Normal gives a completely wrong answer with no error message.
3. **Premature rounding in chained parts.** Rounding an intermediate probability (e.g. to 2 decimal places) before multiplying by n to find an expected count can introduce a large error in the final answer. Always carry the full unrounded GDC value through to the next step.
4. **Forgetting the complement for "greater than" probabilities.** Using Normal CDF with lower $= a$ and upper $= 10^{99}$ is correct, but students sometimes instead compute $P(X < a)$ and forget to subtract from 1, or subtract from the wrong value.
5. **Assertive conclusions after a probability comparison.** A conclusion such as "therefore it is certain that …" is wrong. Probability-based conclusions must be non-assertive: e.g. "there is evidence to suggest …" or "it is likely that …". The R1 mark requires the comparison of values first, then a hedged contextual statement.
6. **Treating the expected count as exact without appropriate rounding.** An expected count of people, items, or events must be rounded to a whole number (or left as a decimal only if the question says "estimate"). Writing 17.3 students as the final answer to "how many students do you expect" loses the accuracy mark.

Section A — Normal Distribution **EASY** (10 questions $\times$ 3 marks = 30 marks)

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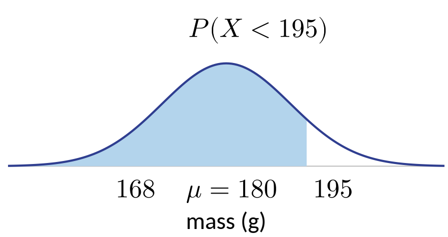
1. **EASY** **Calculator** **[3 marks]**

The mass of apples harvested at a farm in New Zealand is normally distributed with mean $\mu = 180$ g and standard deviation $\sigma = 12$ g. Let X be the mass, in grams, of a randomly selected apple.
Find $P(X < 195)$.

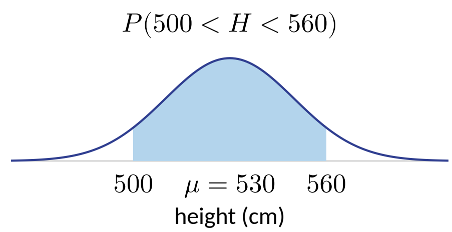


2. **EASY** **Calculator** **[3 marks]**

The time, T minutes, that customers wait in a queue at a bank in Singapore is normally distributed with mean $\mu = 8$ minutes and standard deviation $\sigma = 2$ minutes.
Find $P(T > 11)$.

3. **EASY** **Calculator** **[3 marks]**

The height, H cm, of adult male giraffes in a wildlife reserve in Kenya is normally distributed with mean $\mu = 530$ cm and standard deviation $\sigma = 20$ cm.
Find $P(500 < H < 560)$.



4. **EASY** **Calculator** **[3 marks]**

The volume of coffee, V ml, dispensed by a machine in a café in Italy is normally distributed with mean $\mu = 250$ ml and standard deviation $\sigma = 5$ ml.

(a) Write down $P(V < 250)$. [1]

(b) Find $P(V > 258)$. [2]

5. **EASY** **Calculator** **[3 marks]**

The score, S, achieved by students in a mathematics test at a school in Canada is normally distributed with mean $\mu = 62$ and standard deviation $\sigma = 8$.
Find the value of s such that $P(S < s) = 0.90$.

6. EASY Calculator **[3 marks]**

The daily rainfall, R mm, in a city in Thailand during monsoon season is normally distributed with mean $\mu = 45$ mm and standard deviation $\sigma = 9$ mm.

(a) Find $P(R > 50)$. [2]

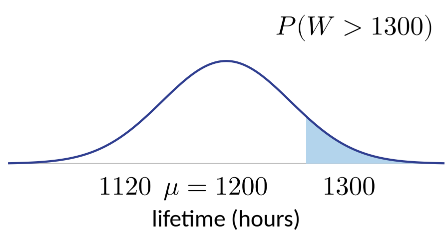
(b) A day is chosen at random. State the probability that the rainfall is less than 45 mm. [1]

7. EASY Calculator **[3 marks]**

The length, L cm, of a species of tropical fish kept in an aquarium in Japan is normally distributed with mean $\mu = 14$ cm and standard deviation $\sigma = 2.5$ cm.
Find the value of l such that $P(L > l) = 0.15$.

8. EASY Calculator **[3 marks]**

The lifetime, W hours, of a brand of light bulb sold in Germany is normally distributed with mean $\mu = 1200$ hours and standard deviation $\sigma = 80$ hours.
There are 500 light bulbs in a warehouse. Estimate the number of bulbs expected to have a lifetime greater than 1300 hours.



9. EASY Calculator **[3 marks]**

The speed, V km/h, of vehicles recorded by a speed camera on a highway in Australia is normally distributed with mean $\mu = 105$ km/h and standard deviation $\sigma = 10$ km/h.
Find $P(95 < V < 115)$.

10. EASY Calculator **[3 marks]**

The mass, M kg, of newborn lambs on a farm in New Zealand is normally distributed with mean $\mu = 4.2$ kg and standard deviation $\sigma = 0.6$ kg.
Find the value of m such that $P(M < m) = 0.25$.

Section B — Medium **MEDIUM** (10 questions $\times$ 4 marks = 40 marks)

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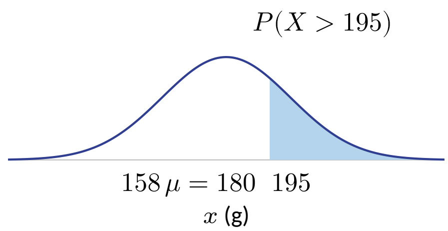
11. **MEDIUM** **Calculator** **[4 marks]**

The masses of avocados sold at a market are normally distributed with mean $\mu = 180$ g and standard deviation $\sigma = 22$ g. Let X be the mass of a randomly selected avocado, in grams.

(a) Find $P(X > 195)$. [2]

(b) In a delivery of 400 avocados, estimate the number with a mass greater than 195 g. [2]



12. **MEDIUM** **Calculator** **[4 marks]**

The daily high temperature in a coastal city during summer is normally distributed with mean $27\,^\circ$C and standard deviation $3.5\,^\circ$C. Let T be the daily high temperature, in $^\circ$C.

(a) Find $P(24 < T < 32)$. [2]

(b) Find the temperature t such that $P(T > t) = 0.10$. [2]

13. MEDIUM Calculator **[4 marks]**

The time, in minutes, that customers spend waiting at a café before being served is normally distributed with mean μ and standard deviation 2.4 min. It is known that $P(\text{wait} < 3) = 0.2090$.
Find the value of μ.

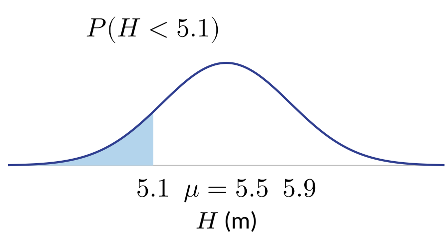
14. MEDIUM Calculator **[4 marks]**

The heights of adult male giraffes in a wildlife reserve are normally distributed with mean 5.5 m and standard deviation 0.35 m. Let H be the height of a randomly selected giraffe, in metres.

(a) Write down $P(H < 5.5)$. [1]

(b) Find $P(H < 5.1)$. [2]

(c) Find $P(5.1 < H < 5.9)$. [1]



15. MEDIUM Calculator **[4 marks]**

The scores on a standardised geography test are normally distributed with mean 62 and standard deviation σ. It is known that 15% of students score more than 80.

(a) Find the value of σ. [3]

(b) Find the probability that a randomly selected student scores less than 50. [1]

16. MEDIUM Calculator **[4 marks]**

The volumes of juice dispensed by a machine into bottles are normally distributed with mean 502 ml and standard deviation 4 ml. A bottle is rejected if it contains less than 494 ml or more than 510 ml.

(a) Find the probability that a randomly selected bottle is rejected. [3]

(b) Find the probability that a randomly selected bottle is accepted. [1]

17. MEDIUM Calculator **[4 marks]**

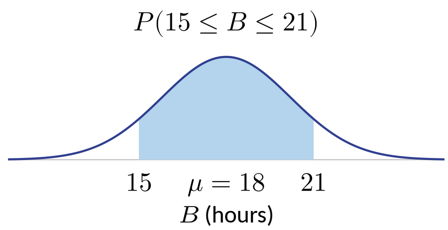
The lengths of fish caught in a lake are normally distributed with mean 38 cm and standard deviation 6.5 cm. Fishing regulations require that any fish shorter than L cm must be returned to the lake. The probability that a randomly caught fish must be returned is 0.05.
Find the value of L.

18. MEDIUM Calculator **[4 marks]**

The battery life of a brand of wireless headphones is normally distributed with mean 18 hours and standard deviation 2.2 hours. Let B be the battery life of a randomly selected pair of headphones, in hours.

(a) Find $P(15 \leq B \leq 21)$. [2]

(b) Find the battery life b such that 20% of headphones last longer than b hours. [2]



19. MEDIUM Calculator **[4 marks]**

The ages of participants registered for a community marathon are normally distributed with mean 34 years and standard deviation 8 years. A participant is classified as a "junior" if they are under 22 years old, and as a "senior" if they are over 50 years old.

(a) Find the probability that a randomly selected participant is classified as a junior. [2]

(b) There are 650 participants in total. Estimate the number who are classified as seniors. [2]

20. MEDIUM Calculator **[4 marks]**

The diameters of bolts produced in a factory are normally distributed with mean 12 mm and standard deviation σ mm. A bolt is defective if its diameter is less than 11 mm or greater than 13 mm. The probability that a bolt is defective is 0.0456.

(a) Find the value of σ. [3]

(b) Find the probability that a bolt has a diameter greater than 12.5 mm. [1]

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)

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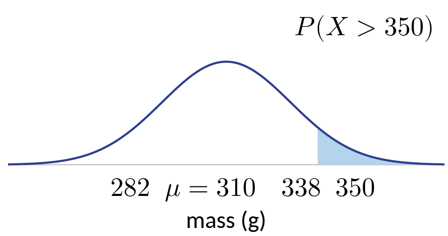
21. **HARD** **Calculator** **[5 marks]**

The masses of mangoes harvested at a farm are normally distributed with mean $\mu = 310$ g and standard deviation $\sigma = 28$ g. A mango is classified as "premium" if its mass exceeds 350 g.

(a) Find the probability that a randomly selected mango is premium. [1]

(b) A crate contains 120 mangoes. Estimate the number of premium mangoes in the crate. [2]

(c) The farmer changes the harvesting process so that the mean mass increases to $\mu = 325$ g while σ remains 28 g. Determine whether the expected number of premium mangoes in a crate of 120 increases by more than 10. Justify your answer. [2]



22. HARD Calculator **[5 marks]**

The daily rental revenue X (in USD) at a city apartment follows a normal distribution with mean μ USD and standard deviation $\sigma = 45$ USD. It is known that $P(X < 180) = 0.159$.

(a) Show that $\mu = 225$ USD. [2]

(b) Find $P(180 < X < 270)$. [2]

(c) The owner considers a day "exceptional" if revenue exceeds 300 USD. Find the probability that, in a randomly chosen week of 7 days, at least 2 days are exceptional. [1]

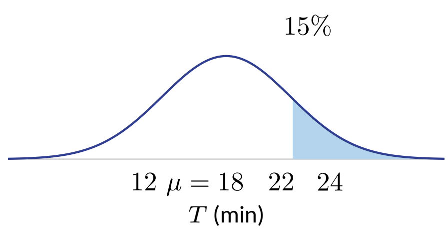
23. HARD Calculator **[5 marks]**

The time, T minutes, taken by students to complete a logic puzzle is normally distributed with mean $\mu = 18$ min and standard deviation σ min. It is given that 15% of students take more than 24 minutes.

(a) Show that $\sigma = 5.79$ minutes (correct to 3 significant figures). [2]

(b) Hence find $P(12 < T < 22)$. [2]

(c) A student is chosen at random. Given that the student took more than 18 minutes, find the probability that they took more than 24 minutes. [1]



24. HARD Calculator **[5 marks]**

A factory fills bottles of olive oil. The volume V ml in a bottle is normally distributed with mean $\mu = 502$ ml and standard deviation $\sigma = 4$ ml. A bottle is rejected if its volume is less than 494 ml or greater than 510 ml.

(a) Find the probability that a randomly selected bottle is rejected. [2]

(b) In a production run of 2000 bottles, estimate the number of bottles that are rejected. [1]

(c) The factory adjusts the machine so that μ changes but σ remains 4 ml. Find the value of μ such that only 1% of bottles have volume less than 494 ml. [2]

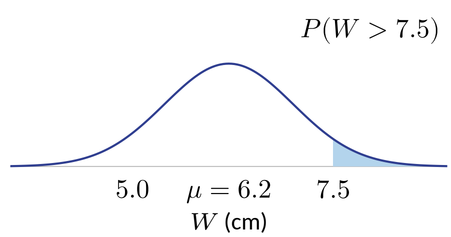
25. HARD Calculator **[5 marks]**

The wingspan, W cm, of a species of butterfly is normally distributed with mean 6.2 cm and standard deviation 0.8 cm.

(a) Find $P(W > 7.5)$. [1]

(b) A researcher collects 200 butterflies. Estimate the number with wingspan between 5.0 cm and 7.5 cm. [2]

(c) The researcher defines a butterfly as "rare" if its wingspan places it in the top 3% of the distribution. Find the minimum wingspan for a butterfly to be classified as rare. [2]



26. HARD Calculator **[5 marks]**

The scores, S, achieved by candidates in a professional examination are normally distributed. The pass mark is 50. It is known that 10% of candidates score below 38 and 5% score above 72.

(a) Write down two equations in μ and σ. [1]

(b) Hence find μ and σ. [2]

(c) Find the probability that a randomly selected candidate passes the examination. [2]

27. HARD Calculator **[5 marks]**

A soft-drink machine dispenses liquid whose volume V ml is normally distributed with mean $\mu = 355$ ml and standard deviation $\sigma = 6$ ml. A cup overflows if it receives more than 368 ml.

(a) Find the probability that a randomly chosen cup overflows. [1]

(b) The machine is recalibrated so that the probability of overflow is reduced to exactly 0.02. The standard deviation remains 6 ml. Find the new mean μ^*. [2]

(c) With the original settings ($\mu = 355$, $\sigma = 6$), find the volume v such that 90% of cups receive at least v ml. [2]

28. HARD Calculator **[5 marks]**

The concentration C (mg/L) of a medication in a patient's bloodstream t hours after administration is modelled by $C(t) = 12t\,e^{-0.5t}$ for $0 \leq t \leq 10$. Independent of this model, measurement errors mean that recorded concentrations are normally distributed about the true value with standard deviation 0.35 mg/L. At $t = 3$ hours the true concentration is $C(3) = 12(3)e^{-1.5}$.

(a) Show that $C(3) = 8.03$ mg/L correct to 3 significant figures. [1]

(b) A recorded reading at $t = 3$ is modelled as $R \sim N(8.03, 0.35^2)$. Find $P(R > 8.5)$. [2]

(c) Find the recorded value r such that only 5% of readings exceed r. [2]

29. HARD Calculator **[5 marks]**

A quality-control engineer tests the breaking strength B kN of steel cables. Breaking strengths follow a normal distribution with mean $\mu = 84$ kN and standard deviation $\sigma = 5$ kN. A cable fails inspection if its breaking strength is below 75 kN.

(a) Find the probability that a randomly selected cable fails inspection. [1]

(b) A batch of 15 cables is tested. Let F be the number of cables that fail inspection. State the distribution of F. [1]

(c) Find the probability that at most 2 cables in the batch fail inspection. [2]

(d) Determine whether it is more likely that exactly 1 cable or exactly 2 cables fail inspection. Justify your answer. [1]

30. **HARD** **Calculator** **[5 marks]**

The heights H cm of adult males in a country are normally distributed with mean 175 cm and standard deviation 7 cm. A doorframe is built to a height d cm so that no more than 2% of adult males need to duck.

(a) Find the minimum integer value of d such that the proportion of adult males who need to duck is at most 2%. [3]

(b) An architect checks two independently chosen adult males. Find the probability that both must duck if $d = 189$ cm. [2]

Worked Solutions

Q1 — Normal Probability Left Tail EASY

We require the probability that a randomly chosen apple has mass less than 195 g, which is the area to the left of 195 under the normal curve with $\mu = 180$, $\sigma = 12$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= -10^{99}$, upper $= 195$, $\mu = 180$, $\sigma = 12$ **M1**

Unrounded value: $0.89435\ldots$
$P(X < 195) = 0.894$ **A2**
Warning: Make sure the lower bound is set to a very large negative number (e.g. -10^{99}), not zero, otherwise the GDC will give an incorrect answer.

Q2 — Normal Probability Right Tail EASY

We require the probability that waiting time exceeds 11 minutes, which is the area to the right of 11 under the normal curve with $\mu = 8$, $\sigma = 2$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 11$, upper $= 10^{99}$, $\mu = 8$, $\sigma = 2$ **M1**

Unrounded value: $0.066807\ldots$
$P(T > 11) = 0.0668$ **A2**
Warning: This is a right-tail probability; set the upper bound to a very large positive number (e.g. 10^{99}), not a finite value, to capture the entire right tail.

Q3 — Normal Probability Central Region EASY

We require the probability that a giraffe's height lies between 500 cm and 560 cm, the area between these two values under the normal curve with $\mu = 530$, $\sigma = 20$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 500$, upper $= 560$, $\mu = 530$, $\sigma = 20$ **M1**

Unrounded value: $0.86638\ldots$
$P(500 < H < 560) = 0.866$ **A2**
Warning: Both boundaries must be entered correctly; swapping or omitting either boundary will give the wrong region.

Q4 — Symmetry and Right-Tail Normal EASY

(a) Since 250 is the mean of the distribution, exactly half the area lies below it. By symmetry:
$P(V < 250) = 0.5$ **A1**

(b) We require the probability that the dispensed volume exceeds 258 ml.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 258$, upper $= 10^{99}$, $\mu = 250$, $\sigma = 5$ **M1**

Unrounded value: $0.054799\ldots$
$P(V > 258) = 0.0548$ **A1**
Warning: Part (a) requires no GDC calculation; the answer follows directly from the symmetry property $P(X < \mu) = 0.5$ for any normal distribution.

Q5 — Inverse Normal Left Tail EASY

We need the value s below which 90% of scores fall — a left-tail inverse normal problem with cumulative probability 0.90.
Setting up: $P(S < s) = 0.90$ with $\mu = 62$, $\sigma = 8$. **M1**
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ invNorm, area $= 0.90$, $\mu = 62$, $\sigma = 8$, LEFT tail
Unrounded value: $72.256\ldots$
$s = 72.3$ **A2**
Warning: The area entered in invNorm is always the area to the LEFT of the required value; since $P(S < s) = 0.90$, the tail area is already 0.90 and no subtraction from 1 is needed.

Q6 — Right-Tail Normal and Symmetry EASY

(a) We require the probability that daily rainfall exceeds 50 mm.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 50$, upper $= 10^{99}$, $\mu = 45$, $\sigma = 9$ **M1**
Unrounded value: $0.28948\ldots$
$P(R > 50) = 0.289$ **A1**
(b) Since 45 mm is the mean of the distribution, by symmetry exactly half the area lies below it:
$P(R < 45) = 0.5$ **A1**
Warning: Part (b) is a symmetry result requiring no GDC; any student who attempts to use normalcdf here wastes time and risks a rounding error.

Q7 — Inverse Normal Right Tail EASY

We need the value l above which 15% of fish lengths lie. Since $P(L > l) = 0.15$, the area to the LEFT of l is $1 - 0.15 = 0.85$.
Setting up: $P(L < l) = 0.85$ with $\mu = 14$, $\sigma = 2.5$. **M1**
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ invNorm, area $= 0.85$, $\mu = 14$, $\sigma = 2.5$, LEFT tail
Unrounded value: $16.591\ldots$
$l = 16.6$ cm **A2**
Warning: The given probability is a RIGHT-tail probability; you must subtract from 1 to obtain the left-tail area before entering it into invNorm, otherwise the GDC returns the wrong boundary.

Q8 — Expected Count from Normal Probability EASY

First find the probability that a single bulb has lifetime greater than 1300 hours.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 1300$, upper $= 10^{99}$, $\mu = 1200$, $\sigma = 80$ **M1**

Unrounded value: $P(W > 1300) = 0.10565\ldots$
Multiply by the total number of bulbs to estimate the expected count:
$500 \times 0.10565\ldots = 52.82\ldots \approx 53$ bulbs **A1A1**

Warning: Round the final expected count to the nearest whole number since it represents a number of physical bulbs; do not leave the answer as a decimal.

Q9 — Normal Probability Central Region EASY

We require the probability that a vehicle's recorded speed lies between 95 km/h and 115 km/h.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 95$, upper $= 115$, $\mu = 105$, $\sigma = 10$ **M1**

Unrounded value: $0.68269\ldots$
$P(95 < V < 115) = 0.683$ **A2**

Warning: Note that 95 and 115 are each exactly one standard deviation from the mean; the answer is the familiar 68.3% empirical-rule result, but the GDC value must still be shown as evidence.

Q10 — Inverse Normal Lower Quartile EASY

We need the value m below which 25% of lamb masses lie — a left-tail inverse normal problem with cumulative probability 0.25.
Setting up: $P(M < m) = 0.25$ with $\mu = 4.2$, $\sigma = 0.6$. **M1**

GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ invNorm, area $= 0.25$, $\mu = 4.2$, $\sigma = 0.6$, LEFT tail
Unrounded value: $3.7953\ldots$
$m = 3.80$ kg **A2**

Warning: The area 0.25 is entered directly as the left-tail area since $P(M < m) = 0.25$; no subtraction from 1 is required here.

Q11 — Normal Probability and Expected Count MEDIUM

(a) To find $P(X > 195)$, we use the upper-tail area with $\mu = 180$, $\sigma = 22$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 195$, upper $= 10^{99}$, $\mu = 180$, $\sigma = 22$ **M1**

Unrounded value: $0.24768\ldots$
$P(X > 195) = 0.248$ **A1**

(b) The expected number of avocados above 195 g is found by multiplying the unrounded probability from (a) by the total number in the delivery.
$400 \times 0.24768\ldots = 99.07\ldots \approx 99$ avocados **A1ft**

Warning: Use the unrounded probability from part (a) in this multiplication; the unrounded value here is $0.24768\ldots$, which rounds to 0.248 at 3 s.f. — always carry the full GDC value forward rather than a partially-rounded intermediate figure.

Q12 — Normal Interval Probability and Inverse Normal MEDIUM

(a) To find $P(24 < T < 32)$, we compute the central area under the normal curve with $\mu = 27$, $\sigma = 3.5$.
GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 24$, upper $= 32$, $\mu = 27$, $\sigma = 3.5$ **M1**

Unrounded value: $0.72775\ldots$
$P(24 < T < 32) = 0.728$ **A1**

(b) We need t such that $P(T > t) = 0.10$; the left-tail area entering invNorm is $1 - 0.10 = 0.90$.
GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.90$ (LEFT tail), $\mu = 27$, $\sigma = 3.5$ **M1**

$t = 31.485\ldots \approx 31.5\,^\circ$C **A1**

Warning: The question asks for $P(T > t) = 0.10$, which is the upper tail; entering area $= 0.10$ into invNorm gives the lower 10% threshold ($22.5\,^\circ$C) instead — the wrong answer.

Q13 — Unknown Mean from One Probability Condition MEDIUM

We are told $P(\text{wait} < 3) = 0.2090$ with $\sigma = 2.4$. We set up a normalcdf equation in the unknown μ and solve using the GDC solver.
The equation to solve is: normalcdf$(-10^{99}, 3, \mu, 2.4) = 0.2090$ **M1**

GDC: Equation Solver (or Numerical Solver), define $f(\mu) =$ normalcdf$(-10^{99}, 3, \mu, 2.4) - 0.2090 = 0$, solve for μ **M1**

$\mu = 4.9438\ldots \approx 4.94$ min **A2**

Warning: Standardising by hand gives $z =$ invNorm$(0.2090) \approx -0.8098$, so $\mu = 3 - 2.4 \times (-0.8098) = 3 + 1.944 = 4.944 \approx 4.94$. Do not round the z-value too early, and be careful with the final addition — it is easy to mis-add $3 + 1.944$ and lose the third significant figure.

Q14 — Symmetry and Tail Probability MEDIUM

(a) Because 5.5 m is the mean of a normal distribution, exactly half the area lies below the mean.
$P(H < 5.5) = 0.5$ **A1**

(b) We find the lower-tail probability below 5.1 m.
GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= -10^{99}$, upper $= 5.1$, $\mu = 5.5$, $\sigma = 0.35$ **M1**

$P(H < 5.1) = 0.12655\ldots \approx 0.127$ **A1**

(c) By symmetry of the normal curve about $\mu = 5.5$, the interval $(5.1, 5.9)$ is symmetric ($5.9 = 5.5 + 0.4$ and $5.1 = 5.5 - 0.4$), so
$P(5.1 < H < 5.9) = 1 - 2 \times P(H < 5.1) = 1 - 2(0.12655\ldots) = 0.74690\ldots \approx 0.747$ **A1ft**

Warning: Use the unrounded value from (b) when computing $1 - 2 \times 0.12655\ldots = 0.747$; rounding 0.12655 to 3 s.f. first and then doubling it loses the third significant figure of the final answer — always carry the unrounded value through chained parts.

Q15 — Unknown Standard Deviation from Upper-Tail Probability MEDIUM

(a) We are told $P(\text{score} > 80) = 0.15$ with $\mu = 62$, and we need to find σ. We set up the normalcdf equation in σ and use the GDC solver.
The equation to solve is: $\text{normalcdf}(80, 10^{99}, 62, \sigma) = 0.15$ **M1**

GDC: Numerical Solver, define $f(\sigma) = \text{normalcdf}(80, 10^{99}, 62, \sigma) - 0.15 = 0$, solve for $\sigma > 0$ **M1**

$\sigma = 17.367\ldots \approx 17.4$ **A1**

(b) Using the unrounded value of σ found in (a), we find the probability of scoring below 50.
GDC: $\text{normalcdf}(-10^{99}, 50, 62, 17.367\ldots) = 0.24480\ldots \approx 0.245$ **A1ft**

Warning: Use the full unrounded σ from part (a) in the GDC for part (b); the equation solves to $\sigma \approx 17.4$ (since invNorm$(0.85) \approx 1.0364$ and $18/1.0364 \approx 17.4$) — a common slip is using $z =$ invNorm(0.15) (the wrong tail) instead of $z =$ invNorm(0.85), which gives a badly wrong σ.

Q16 — Rejection Probability as Sum of Two Tails MEDIUM

(a) A bottle is rejected if its volume falls outside $[494, 510]$. We compute the two rejection tails separately and add them.
Lower tail: GDC: $\text{normalcdf}(-10^{99}, 494, 502, 4) = 0.02275\ldots$ **M1**

Upper tail: GDC: $\text{normalcdf}(510, 10^{99}, 502, 4) = 0.02275\ldots$
$P(\text{rejected}) = 0.02275\ldots + 0.02275\ldots = 0.04550\ldots \approx 0.0455$ **A1**

Alternatively: $1 - \text{normalcdf}(494, 510, 502, 4) = 1 - 0.95449\ldots = 0.04550\ldots \approx 0.0455$ **A1**

(b) The probability of acceptance is the complement of the rejection probability.
$P(\text{accepted}) = 1 - 0.04550\ldots = 0.95449\ldots \approx 0.954$ **A1ft**

Warning: Note that $\mu = 502$ is centred exactly midway between 494 and 510 (both 8 ml from the mean), so the two tails are equal here — but always verify this rather than assuming it from the limits.

Q17 — Inverse Normal Lower-Tail Threshold MEDIUM

We need the length L such that $P(H < L) = 0.05$, where $H \sim N(38, 6.5^2)$ — a lower-tail inverse normal problem.
We require the value at the left tail with area 0.05. **M1**

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.05$ (LEFT tail), $\mu = 38$, $\sigma = 6.5$ **M1**

$L = 27.308\ldots \approx 27.3$ cm **A2**

Warning: The 5% of fish that must be returned are the shortest fish, so the threshold is in the left tail.

Entering area $= 0.95$ in invNorm gives the right-tail threshold (48.7 cm), which is the wrong end of the distribution.

Q18 — Normal Interval and Inverse Normal Upper Tail MEDIUM

(a) To find $P(15 \leq B \leq 21)$, we compute the central area for $B \sim N(18, 2.2^2)$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 15$, upper $= 21$, $\mu = 18$, $\sigma = 2.2$ **M1**

Unrounded value: $0.82732\ldots$
$P(15 \leq B \leq 21) = 0.827$ **A1**

(b) We need b such that $P(B > b) = 0.20$; the left-tail area entering invNorm is $1 - 0.20 = 0.80$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ invNorm, area $= 0.80$ (LEFT tail), $\mu = 18$, $\sigma = 2.2$ **M1**

$b = 19.850\ldots \approx 19.9$ hours **A1**

Warning: The question states that 20% of headphones last longer than b, meaning b is in the upper portion of the distribution. Entering area $= 0.20$ into invNorm gives the lower 20% threshold (16.2 hours) — the wrong answer.

Q19 — Two-Tail Probabilities and Expected Count MEDIUM

(a) A junior participant is under 22 years old. We find the lower-tail probability for $A \sim N(34, 8^2)$.
GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= -10^{99}$, upper $= 22$, $\mu = 34$, $\sigma = 8$ **M1**

$P(A < 22) = 0.06681\ldots \approx 0.0668$ **A1**

(b) A senior is over 50 years old. We first find $P(A > 50)$, then multiply by the total number of participants.
GDC: normalcdf$(50, 10^{99}, 34, 8) = 0.02275\ldots$
Expected number of seniors $= 650 \times 0.02275\ldots = 14.788\ldots \approx 15$ participants **A2**

Warning: The two age-group boundaries (22 and 50) are not symmetric about $\mu = 34$ ($22 = 34 - 12$, $50 = 34 + 16$), so the two probabilities are not equal — do not assume symmetry and copy the answer from (a).

Q20 — Unknown Standard Deviation from Two-Tail Probability MEDIUM

(a) The defect condition is $|d - 12| > 1$, i.e. diameter outside $(11, 13)$. Because $\mu = 12$ is centred in the interval, both tails are equal, so $P(\text{defective}) = P(D < 11) + P(D > 13) = 0.0456$.
By symmetry, $P(D < 11) = 0.0228$. We set up normalcdf$(-10^{99}, 11, 12, \sigma) = 0.0228$ and solve for σ. **M1**

GDC: Numerical Solver, $f(\sigma) =$ normalcdf$(-10^{99}, 11, 12, \sigma) - 0.0228 = 0$, solve for $\sigma > 0$ **M1**

$\sigma = 0.50023\ldots \approx 0.500$ mm **A1**

(b) Using $\mu = 12$ and the unrounded $\sigma = 0.50023\ldots$, we find the upper-tail probability above 12.5 mm.
GDC: normalcdf$(12.5, 10^{99}, 12, 0.50023\ldots) = 0.15877\ldots \approx 0.159$ **A1ft**

Warning: The symmetry argument ($P(D < 11) = P(D > 13) = 0.0228$) only holds because $\mu = 12$ is exactly midway between 11 and 13; if the mean were not centred, you would need to solve the full two-tail equation without invoking symmetry. Check the third significant figure of σ carefully — it is easy to round the solver's output the wrong way.

Q21 — Premium Mango Proportion and Expected Count HARD

(a) We need $P(X > 350)$ where $X \sim N(310, 28^2)$.
GDC: Statistics → Distributions → Normal CDF, lower $= 350$, upper $= 10^{99}$, $\mu = 310$, $\sigma = 28$.
$P(X > 350) = 0.07656\ldots \approx 0.0766$ **A1**

(b) The expected number of premium mangoes is found by multiplying the sample size by the unrounded probability from part (a).
(M1): Expected number $= 120 \times 0.07656\ldots$ **(M1)**

$= 9.187\ldots \approx 9.19$ mangoes **A1ft**

(c) We repeat the calculation for the new mean $\mu = 325$ g and compare the increase with 10.
GDC: Normal CDF, lower $= 350$, upper $= 10^{99}$, $\mu = 325$, $\sigma = 28$: $P(X > 350) = 0.18597\ldots$
New expected number $= 120 \times 0.18597\ldots = 22.316\ldots$
Increase $= 22.316\ldots - 9.187\ldots = 13.13\ldots > 10$ **R1**
Therefore the expected number of premium mangoes does increase by more than 10. **A1**

Warning: Using a rounded probability instead of the unrounded value in part (b) and then carrying that rounded figure into part (c) will accumulate error and may give an incorrect conclusion at the comparison step.

Q22 — Apartment Revenue: Reversal Then Fusion HARD

(a) We are told $P(X < 180) = 0.159$ with $\sigma = 45$; we must recover μ and verify it equals 225.
Setting up: $P(X < 180) = 0.159$ with $X \sim N(\mu, 45^2)$.
GDC: Statistics → Distributions → Inverse Normal, area $= 0.159$, LEFT tail, $\mu = 0$, $\sigma = 1$ gives $z = -0.99958\ldots \approx -1.000$.
Standardising: $\dfrac{180 - \mu}{45} = -1.000$, so $180 - \mu = -45.0$, giving $\mu = 225$. **M1**

Alternative (solver route): GDC solver on normalcdf$(-10^{99}, 180, \mu, 45) = 0.159$ gives $\mu = 225.0$
$\mu = 225$ USD **AG**

(b) With $X \sim N(225, 45^2)$ we find $P(180 < X < 270)$.
GDC: Normal CDF, lower $= 180$, upper $= 270$, $\mu = 225$, $\sigma = 45$. **M1**

$P(180 < X < 270) = 0.68269\ldots \approx 0.683$ **A1**

(c) First find $p = P(X > 300)$: GDC Normal CDF, lower $= 300$, upper $= 10^{99}$, $\mu = 225$, $\sigma = 45$ gives $p = 0.04779\ldots$.
Let $Y \sim B(7, 0.04779\ldots)$. $P(Y \geq 2) = 1 - P(Y \leq 1)$.
GDC: Binomial CDF, $n = 7$, $p = 0.04779\ldots$, $x = 1$: $P(Y \leq 1) = 0.95915\ldots$

$P(Y \geq 2) = 1 - 0.95915\ldots = 0.04085\ldots \approx 0.0408$ **A1**

Warning: 180 is exactly one standard deviation below 225 ($225 - 45 = 180$), giving the classic $z = -1$ result $P(Z < -1) = 0.159$; this is why $P(X < 180)$ must be 0.159, not 0.0668, for $\mu = 225$ to hold. Also check the binomial step carefully: $P(Y \geq 2) = 1 - P(Y \leq 1)$, not $1 - P(Y \leq 2)$.

Q23 — Logic Puzzle Times: Find Sigma Then Conditional Probability HARD

(a) We know $P(T > 24) = 0.15$ with $\mu = 18$; we must find σ and verify $\sigma = 5.79$ min.
GDC solver: solve normalcdf$(24, 10^{99}, 18, \sigma) = 0.15$ for σ. **M1**

Unrounded value: $\sigma = 5.7891\ldots$ min
$\sigma = 5.79$ min (3 s.f.) **AG**

(b) Using $T \sim N(18, 5.79^2)$ (unrounded $5.7891\ldots$), we find $P(12 < T < 22)$.
GDC: Normal CDF, lower $= 12$, upper $= 22$, $\mu = 18$, $\sigma = 5.7891\ldots$. **M1**

$P(12 < T < 22) = 0.60520\ldots \approx 0.605$ **A1**

(c) We want $P(T > 24 \mid T > 18)$. Since the normal distribution is symmetric about the mean, $P(T > 18) = 0.5$.

$$P(T > 24 \mid T > 18) = \frac{P(T > 24 \cap T > 18)}{P(T > 18)} = \frac{P(T > 24)}{P(T > 18)} = \frac{0.15}{0.5} = 0.30$$ **A1**

Warning: Using the rounded $\sigma = 5.79$ instead of the full unrounded solver value $5.7891\ldots$ in part (b) can shift the third significant figure — always carry the unrounded value forward.

Q24 — Olive Oil Bottles: Two-Tail Rejection and Reversal HARD

(a) We need $P(V < 494) + P(V > 510)$ with $V \sim N(502, 4^2)$.
GDC: Normal CDF for left tail: lower $= -10^{99}$, upper $= 494$, $\mu = 502$, $\sigma = 4$: $P(V < 494) = 0.02275\ldots$ **(M1)**

GDC: Normal CDF for right tail: lower $= 510$, upper $= 10^{99}$: $P(V > 510) = 0.02275\ldots$
$P(\text{rejected}) = 0.02275\ldots + 0.02275\ldots = 0.04550\ldots \approx 0.0455$ **A1**

(b) Expected number rejected $= 2000 \times 0.04550\ldots = 91.0\ldots \approx 91$ bottles. **A1**

(c) We now require $P(V < 494) = 0.01$ with $\sigma = 4$ and unknown μ^*.
GDC solver: normalcdf$(-10^{99}, 494, \mu^*, 4) = 0.01$. **M1**

$\mu^* = 494 - 4 \times \text{invNorm}(0.01) = 494 + 4 \times 2.3263\ldots = 503.305\ldots$
$\mu^* = 503$ ml (3 s.f.) **A1**

Warning: This question uses a TWO-sided rejection region in part (a); adding only one tail gives half the correct rejection probability. In part (c), the left-tail condition means the mean must be above 494 ml — mistaking the tail direction gives $\mu^* \approx 484.7$ ml.

Q25 — Butterfly Wingspan Proportion Count and Top-3% Threshold HARD

(a) With $W \sim N(6.2, 0.8^2)$, evaluate $P(W > 7.5)$.
GDC: Normal CDF, lower $= 7.5$, upper $= 10^{99}$, $\mu = 6.2$, $\sigma = 0.8$.
$P(W > 7.5) = 0.05208\ldots \approx 0.0521$ **A1**

(b) First find $P(5.0 < W < 7.5)$: GDC Normal CDF, lower $= 5.0$, upper $= 7.5$, $\mu = 6.2$, $\sigma = 0.8$. **M1**

$P(5.0 < W < 7.5) = 0.89134\ldots$
Expected count $= 200 \times 0.89134\ldots = 176.22\ldots \approx 176$ butterflies. **A1**

(c) The top 3% means $P(W > w) = 0.03$, so we need the 97th percentile — the RIGHT tail, so the inverse-normal input is the LEFT-tail area $= 0.97$.
GDC: Statistics $\to$ Distributions $\to$ Inverse Normal, area $= 0.97$, $\mu = 6.2$, $\sigma = 0.8$, LEFT tail. **M1**

$w = 7.7046\ldots \approx 7.70$ cm **A1**

Warning: "Top 3%" is the RIGHT tail, but inverse-normal requires the LEFT-tail cumulative area $= 0.97$, *not* 0.03. *Entering* 0.03 *gives the bottom 3% threshold (*≈ 4.70 *cm). Also check part (b) arithmetic carefully:* $200 \times 0.89134\ldots = 176.2$, *which rounds to 176 butterflies.*

Q26 — Exam Scores: Two-Condition Parameter Hunt HARD

(a) Let $S \sim N(\mu, \sigma^2)$. The two probability conditions translate directly to:
$P(S < 38) = 0.10$ and $P(S > 72) = 0.05$ **A1**

(b) Converting each condition to a z-score equation using GDC inverse-normal:

From $P(S < 38) = 0.10$: GDC invNorm, area $= 0.10$, LEFT tail gives $z_1 = -1.2816\ldots$, so $\dfrac{38-\mu}{\sigma} = -1.2816\ldots$ **M1**

From $P(S > 72) = 0.05$: GDC invNorm, area $= 0.95$, LEFT tail gives $z_2 = 1.6449\ldots$, so $\dfrac{72-\mu}{\sigma} = 1.6449\ldots$

Solving simultaneously: subtracting gives $\sigma(1.2816 + 1.6449) = 34$, $\sigma = \dfrac{34}{2.9265\ldots} = 11.62\ldots \approx 11.6$.
Then $\mu = 38 + 1.2816\ldots \times 11.62\ldots = 52.89\ldots \approx 52.9$.
$\mu = 52.9$, $\sigma = 11.6$ (3 s.f.) **A1**

(c) With $S \sim N(52.9, 11.6^2)$ (use unrounded values $\mu = 52.89\ldots$, $\sigma = 11.62\ldots$), find $P(S > 50)$.
GDC: Normal CDF, lower $= 50$, upper $= 10^{99}$, $\mu = 52.89\ldots$, $\sigma = 11.62\ldots$ **M1**

$P(S > 50) = 0.59820\ldots \approx 0.598$ **A1**

Warning: Use the full unrounded μ *and* σ *throughout; rounding to* $\mu = 52.9$ *and* $\sigma = 11.6$ *before part (c) can shift the third significant figure of the final probability.*

Q27 — Drinks Machine Overflow Probability Reversal and Lower Percentile HARD

(a) With $V \sim N(355, 6^2)$, find $P(V > 368)$.
GDC: Normal CDF, lower $= 368$, upper $= 10^{99}$, $\mu = 355$, $\sigma = 6$.

$P(V > 368) = 0.015130\ldots \approx 0.0151$ **A1**

(b) We require $P(V > 368) = 0.02$ with $\sigma = 6$ and unknown μ^*; this is a rightward-tail reversal.
GDC solver: normalcdf$(368, 10^{99}, \mu^*, 6) = 0.02$. **M1**

Equivalently, 368 is the 98th percentile of $N(\mu^*, 36)$: $\mu^* = 368 - 6 \times \text{invNorm}(0.98) = 368 - 6 \times 2.0537\ldots = 355.68\ldots$
$\mu^* = 355.68\ldots \approx 356$ ml (3 s.f.) **A1**

(c) We want v such that $P(V \geq v) = 0.90$, i.e. $P(V < v) = 0.10$ — the LEFT tail at the 10th percentile.
GDC: Inverse Normal, area $= 0.10$, $\mu = 355$, $\sigma = 6$, LEFT tail. **M1**

$v = 347.31\ldots \approx 347$ ml **A1**

Warning: In part (c), "at least v ml" means the LEFT tail gives $P(V < v) = 0.10$, not $P(V < v) = 0.90$. Entering area $= 0.90$ gives the 90th percentile (≈ 362.7 ml), which answers the wrong question. Also check part (a) to the full 4 s.f. ($0.015130\ldots$) before rounding to 3 s.f.

Q28 — Medication Concentration Show That and Inverse Normal HARD

(a) Evaluate $C(3) = 12(3)e^{-0.5\times 3} = 36e^{-1.5}$.
$36e^{-1.5} = 36 \times 0.22313\ldots = 8.0327\ldots$ **M1**

$= 8.03$ mg/L (3 s.f.) **AG**

(b) With $R \sim N(8.03, 0.35^2)$, find $P(R > 8.5)$.
GDC: Normal CDF, lower $= 8.5$, upper $= 10^{99}$, $\mu = 8.03$, $\sigma = 0.35$. **M1**

$P(R > 8.5) = 0.08966\ldots \approx 0.0897$ **A1**

(c) We want r such that $P(R > r) = 0.05$, i.e. the 95th percentile; the LEFT-tail cumulative area is 0.95.
GDC: Inverse Normal, area $= 0.95$, $\mu = 8.03$, $\sigma = 0.35$, LEFT tail. **M1**

$r = 8.03 + 0.35 \times 1.6449\ldots = 8.6057\ldots \approx 8.61$ mg/L **A1**

Warning: $36e^{-1.5} = 8.0327\ldots$ — check the third significant figure carefully in "show that" questions, since the rounded value here feeds directly into parts (b) and (c).

Q29 — Steel Cables: Normal to Binomial Fusion with Justified Decision HARD

(a) With $B \sim N(84, 5^2)$, find $P(B < 75)$.
GDC: Normal CDF, lower $= -10^{99}$, upper $= 75$, $\mu = 84$, $\sigma = 5$.
$P(B < 75) = 0.03593\ldots \approx 0.0359$ **A1**

(b) Since each cable is tested independently and the probability of failure is constant, $F \sim B(15, 0.03593\ldots)$. **A1**

(c) Find $P(F \leq 2) = P(F = 0) + P(F = 1) + P(F = 2)$.
GDC: Binomial CDF, $n = 15$, $p = 0.03593\ldots$, upper $= 2$. **M1**

$P(F \leq 2) = 0.98474\ldots \approx 0.985$ **A1**

(d) Compute both individual probabilities:
GDC Binomial PDF: $P(F = 1) = 0.32290\ldots \approx 0.323$ and $P(F = 2) = 0.08424\ldots \approx 0.0842$.
Since $P(F = 1) = 0.323 > P(F = 2) = 0.0842$, exactly 1 cable failing is more likely. **R1**

Warning: The probability p for the binomial in part (b) must be the unrounded value from part (a); rounding p to 0.036 before the CDF/PDF calculations introduces errors at the third significant figure of the binomial probabilities.

Q30 — Doorframe Height: Integer Bracketing and Combined Probability HARD

(a) We need the smallest integer d such that $P(H > d) \leq 0.02$, i.e. $P(H \leq d) \geq 0.98$, where $H \sim N(175, 7^2)$.
GDC: Inverse Normal, area $= 0.98$, $\mu = 175$, $\sigma = 7$, LEFT tail gives the continuous threshold: **M1**

$d^* = 175 + 7 \times 2.0537\ldots = 189.376\ldots$ cm **A1**

Since d must be an integer, we check both candidates:
$d = 189$: $P(H > 189) =$ Normal CDF$(189, 10^{99}, 175, 7) = 0.02275\ldots > 0.02$ (fails condition).
$d = 190$: $P(H > 190) =$ Normal CDF$(190, 10^{99}, 175, 7) = 0.01606\ldots \leq 0.02$ (satisfies condition). **A1**

Minimum integer value of d is 190 cm.

(b) With $d = 189$ cm, $P(H > 189) = 0.02275\ldots$ (from above). The two males are chosen independently, so:
$P(\text{both duck}) = (0.02275\ldots)^2 = 0.000518\ldots$ **M1**

$= 5.18 \times 10^{-4}$ **A1**

Warning: The integer-bracketing check in part (a) is mandatory; simply rounding 189.376 down to 189 without verification gives the wrong answer because $d = 189$ does not satisfy $P(H > d) \leq 0.02$.