

IB Math AI SL — Topic 4 Probability Distributions

Discrete Probability Distributions · Expected Values

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50



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Name: _____ Date: _____
Score: _____/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

Key Formulae

- **Sum of all probabilities:** $\sum_{\text{all } x} P(X = x) = 1$
- **Expected value (mean):** $E(X) = \sum_{\text{all } x} x \cdot P(X = x)$
- **Expected value of a function:** $E(g(X)) = \sum_{\text{all } x} g(x) \cdot P(X = x)$
- **Linear transformation of expectation:** $E(aX + b) = aE(X) + b$
- **Complementary probability:** $P(X \geq a) = 1 - P(X \leq a - 1)$
- **Conditional probability:** $P(A | B) = \frac{P(A \cap B)}{P(B)}$
- **Fair game condition:** expected net winnings = 0, i.e. $E(\text{winnings}) = \text{cost of play}$

GDC Tips

- **Storing a distribution and computing $E(X)$:** Enter x -values in List 1 and $P(X = x)$ values in List 2; go to Statistics → 1-Var Stats, set XList=L1, FreqList=L2; read off $\bar{x}$, which equals $E(X)$.
- **Checking the sum-to-1 condition:** With probabilities in List 2, use List → Math → sum(L2) and confirm the result equals 1 before proceeding.
- **Solving for an unknown k :** Enter the sum-to-1 equation symbolically in Algebra → Solve (e.g. solve($0.1 + k + 0.3 + 2k = 1$, k)) to find k without manual algebra.
- **Computing $E(g(X))$:** Define a new list $L3 = g(L1)$ and then $L4 = L3 * L2$; compute sum(L4) to obtain $E(g(X))$ directly.
- **Evaluating cumulative probabilities:** For $P(X \leq a)$, highlight relevant cells in List 2 and use sum; for $P(X \geq a)$ compute 1–sum of the preceding cells.
- **Simultaneous equations for two unknowns:** If sum-to-1 and a given $E(X)$ each yield an equation in p and q , use Algebra → Solve System of Equations → 2 equations, 2 unknowns.

Common Mistakes to Avoid

1. **Forgetting to use sum-to-1 first.** Whenever a probability table contains an unknown k , the very first line of working must set $\sum P(X = x) = 1$ and solve for k . Every subsequent probability read from the table depends on this value; skipping this step invalidates all following parts.
2. **Using an unsolved k in $E(X)$.** Students sometimes substitute the symbolic expression for k into $E(X) = \sum x \cdot P(X = x)$ and then forget to replace k with its numerical value. Always substitute the fully evaluated k before multiplying and summing.
3. **Confusing $E(X)$ with the most likely value.** $E(X)$ is the long-run average, computed as a weighted sum; it need not equal any value in the table and is not necessarily the x with the highest probability. Never read $E(X)$ directly from the table without calculating.
4. **Misreading the fair-game condition.** A game is fair when $E(\text{net gain}) = 0$, meaning the entry fee equals $E(\text{prize})$. A common error is setting the fee equal to $E(X)$ without subtracting the fee itself from each outcome first, or equivalently setting $E(\text{prize}) - \text{fee} = 0$ but then forgetting to check the sign carefully.
5. **Scaling expected winnings incorrectly over N plays.** If $E(\text{net gain per game}) = -0.40$, then over 50 games the expected total loss is $50 \times (-0.40) = -\$20.00$, not $-\$0.40$. Always state clearly whether a computed expectation is per game or total, and multiply explicitly by N .
6. **Accepting a negative or out-of-range probability.** After solving for k , always verify that every individual probability satisfies $0 \leq P(X = x) \leq 1$. If any value of k produces a negative probability or a value exceeding 1, that solution must be rejected and the setup rechecked before continuing.

Section A — Easy **EASY** (10 questions $\times$ 3 marks = 30 marks)



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1. **EASY** **Calculator**

[3 marks]

The discrete random variable X has the following probability distribution, where k is a constant.

x	1	2	3	4
$P(X = x)$	0.2	k	0.35	0.1

Find the value of k .

2. **EASY** **Calculator**

[3 marks]

A bag contains coloured counters. The discrete random variable Y represents the number shown on a randomly selected card, with probability distribution given below, where k is a constant.

y	0	1	2
$P(Y = y)$	0.4	k	0.25

(a) Find the value of k .

(b) Find $E(Y)$.

3. **EASY** **Calculator**

[3 marks]

The discrete random variable W takes values 1, 2 and 3 only. It is known that $P(W = 1) = 0.5$, $P(W = 2) = 0.3$ and $P(W = 3) = 0.2$.

Find $E(W)$.

4. **EASY** **Calculator**

[3 marks]

A spinner at a school fair is divided into four sectors labelled 1, 2, 3 and 4. The probability of landing on each sector is shown in the table below, where k is a constant.

x	1	2	3	4
$P(X = x)$	k	0.3	$2k$	0.1

(a) Find the value of k .

(b) Write down $P(X = 3)$.

5. **EASY** **Calculator**

[3 marks]

The discrete random variable X represents the number of text messages a student sends in one hour. The probability distribution of X is given by

$$P(X = x) = \frac{x}{10}, \quad x = 1, 2, 3, 4.$$

(a) Show that the probabilities sum to 1.

(b) Find $E(X)$.

6. **EASY** **Calculator**

[3 marks]

A discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.05	0.20	0.40	0.25	0.10

(a) Find $E(X)$.

(b) Write down $P(X \geq 3)$.

7. **EASY** **Calculator**

[3 marks]

A prize wheel at a charity event stops on one of three amounts: \$0, \$5 or \$10. The respective probabilities are 0.60, 0.30 and 0.10. Let X represent the prize amount, in dollars, won on a single spin.

Find $E(X)$.

8. **EASY** **Calculator**

[3 marks]

The discrete random variable T takes the values 2, 4 and 6 only, with the following probability distribution, where k is a constant.

t	2	4	6
$P(T = t)$	0.3	k	0.2

(a) Find the value of k .

(b) Find $E(T)$.

9. **EASY** **Calculator**

[3 marks]

The discrete random variable X represents the score on a biased four-sided die with faces numbered 1 to 4. The probabilities are $P(X = 1) = 0.1$, $P(X = 2) = 0.4$, $P(X = 3) = 0.3$ and $P(X = 4) = 0.2$. Find $E(X)$.

10. **EASY** **Calculator**

[3 marks]

The discrete random variable M has probability distribution given by

$$P(M = m) = \frac{k}{m}, \quad m = 1, 2, 4.$$

(a) Find the value of k .

(b) Find $E(M)$.

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)



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11. **MEDIUM** **Calculator** [4 marks]

A school carnival game involves spinning a wheel. The number of tokens, X , a player wins in one spin has the probability distribution shown in the table below.

x	0	1	3	5
$P(X = x)$	0.40	k	$2k$	0.10

(a) Find the value of k . [2]

(b) Find $E(X)$, the expected number of tokens won per spin. [2]

12. **MEDIUM** **Calculator** [4 marks]

A café loyalty card gives customers a random reward each visit. The monetary reward, R (in dollars), has the following probability distribution, where a and b are constants.

r	0	2	5	10
$P(R = r)$	a	0.35	b	0.05

Given that $E(R) = 1.65$, find the values of a and b .

13. MEDIUM Calculator

[4 marks]

A charity raffle sells 200 tickets at \$3 each. There is one prize of \$150, two prizes of \$50, and five prizes of \$10. A player buys one ticket. Let W be the net winnings (prize money minus ticket cost) for the player.

- (a) Write down all possible values of W . [1]
- (b) Find $E(W)$. [2]
- (c) Determine whether buying a ticket is a fair game. Justify your answer. [1]

14. MEDIUM Calculator

[4 marks]

A discrete random variable X has probability distribution defined by

$$P(X = x) = k(x^2 + 1), \quad x \in \{0, 1, 2, 3\}.$$

- (a) Show that $k = \frac{1}{18}$. [2]
- (b) Find $E(X)$. [2]

15. MEDIUM Calculator

[4 marks]

Two fair four-sided dice, each numbered 1 to 4, are rolled. Let M be the minimum of the two values shown.

- (a) Complete the probability distribution table for M . [2]

m	1	2	3	4
$P(M = m)$	$\frac{7}{16}$			$\frac{1}{16}$

- (b) Find $E(M)$. [2]

16. **MEDIUM** **Calculator** [4 marks]

A game at a school fair costs \$4 to play. A player rolls a standard fair six-sided die. If the result is a 6, the player wins \$15. If the result is a 4 or 5, the player wins \$5. Otherwise, the player wins nothing. Let G be the player's net gain (winnings minus entry fee) in one play.

(a) Find $E(G)$. [3]

(b) A student plans to play this game 30 times. Find the student's expected total net gain. [1]

17. **MEDIUM** **Calculator** [4 marks]

The number of goals, X , scored by a school football team in a match has the probability distribution shown.

x	0	1	2	3	4
$P(X = x)$	0.15	0.30	k	0.15	0.05

(a) Find the value of k . [1]

(b) Find $E(X)$. [2]

(c) Interpret the value of $E(X)$ in context. [1]

18. **MEDIUM** **Calculator** [4 marks]

A spinner is divided into four sectors labelled 1, 2, 3, and 4. The probability of landing on sector x is proportional to x (i.e. $P(X = x) = cx$ for some constant c).

(a) Show that $c = 0.1$. [1]

(b) Find $P(X \geq 3)$. [1]

(c) Find $E(X)$. [2]

19. MEDIUM Calculator

[4 marks]

A bag contains 4 red and 6 blue counters. Two counters are drawn without replacement. Let Y be the number of red counters drawn.

(a) Find the probability distribution of Y , showing all probabilities as fractions. [2]

(b) Find $E(Y)$. [2]

20. MEDIUM Calculator

[4 marks]

A discrete random variable X has the probability distribution given below, where p and q are positive constants.

x	1	2	4	8
$P(X = x)$	p	0.20	q	0.10

Given that $E(X) = 3.2$, find the values of p and q .

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)



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21. **HARD** **Calculator** [5 marks]

A board game uses a spinner with four sectors numbered 1, 2, 3 and 4. The probability distribution of the score X on a single spin is given in the table below.

x	1	2	3	4
$P(X = x)$	$2k$	$3k$	$k + 0.1$	0.3

- (a) Show that $k = 0.1$. [2]
- (b) Find $E(X)$. [2]
- (c) A player pays \$1.50 to spin once and receives \$ X in return. Determine whether the game is fair. Justify your answer. [1]

22. **HARD** **Calculator** [5 marks]

A discrete random variable Y has the probability distribution defined by

$$P(Y = y) = c(y^2 + 1), \quad y \in \{0, 1, 2, 3\}.$$

- (a) Find the value of c . [2]
- (b) Find $P(Y \geq 2)$. [1]
- (c) Find $E(Y)$. [2]

23. **HARD** Calculator

[5 marks]

A bag contains 3 red and 2 blue tokens. Two tokens are drawn without replacement. The discrete random variable W represents the number of red tokens drawn.

(a) Show that $P(W = 2) = \frac{3}{10}$. [2]

(b) Complete the probability distribution of W and find $E(W)$. [3]

24. **HARD** Calculator

[5 marks]

Two fair six-sided dice are rolled. The random variable M is the minimum of the two scores shown.

(a) Show that $P(M = 1) = \frac{11}{36}$. [2]

(b) Write down $P(M = 6)$. [1]

(c) Find $E(M)$. [2]

25. **HARD** Calculator

[5 marks]

The probability distribution of a discrete random variable X is given in the table below, where a and b are positive constants.

x	0	1	2	3
$P(X = x)$	a	0.3	b	0.1

Given that $E(X) = 1.2$, find the values of a and b .

26. **HARD** Calculator

[5 marks]

At a school fair, a game involves rolling a single fair six-sided die. A player wins \$5 if the die shows a 6, wins \$2 if the die shows a 4 or 5, and wins nothing otherwise. The cost to play is \$ c .

- Find $E(W)$, where W is the amount won (excluding the entry fee). [2]
- Find the value of c that makes the game fair. [1]
- A player intends to play this game 30 times. Using your value of c from part (b), find the expected total profit (or loss) over 30 plays. [2]

27. **HARD** Calculator

[5 marks]

A discrete random variable X has the probability distribution defined by

$$P(X = x) = k(4 - x), \quad x \in \{0, 1, 2, 3\}.$$

- Find the value of k . [1]
- Find $E(X)$. [2]
- Given that a player pays \$0.80 per game and receives \$ X in return, find the player's expected loss per game. Determine whether a rational player should participate. Justify your answer. [2]

28. **HARD** **Calculator** [5 marks]

A raffle sells 50 tickets at \$4 each. There is one first prize of \$80, two second prizes of \$20 each, and no other prizes. A player buys one ticket. Let G be the net gain to the player (prize money received minus ticket cost).

- (a) Write down the probability distribution of G . [2]
- (b) Find $E(G)$. [2]
- (c) Determine, with justification, whether buying a ticket is a financially rational decision. [1]

29. **HARD** **Calculator** [5 marks]

The discrete random variable X has the following probability distribution, where p and q are positive constants with $p > q$.

x	-1	0	1	2
$P(X = x)$	p	0.25	q	0.05

Given that $E(X) = 0$, find p and q .

30. **HARD** **Calculator** [5 marks]

A factory produces light bulbs. The number of defective bulbs D in a randomly selected box of 3 bulbs has the probability distribution shown below, where m is a constant.

d	0	1	2	3
$P(D = d)$	0.5	m	0.1	0.02

- (a) Find the value of m . [1]
- (b) Find $E(D)$. [2]
- (c) A quality inspector tests 5 independent boxes and considers the process acceptable if the expected total number of defective bulbs across all 5 boxes is less than 2. Determine whether the process is acceptable. Justify your answer. [2]

Mark Schemes

Q1 — Find k from Sum-to-One EASY

Because all probabilities in a distribution must sum to exactly 1, we write the equation and solve for k .

$$0.2 + k + 0.35 + 0.1 = 1 \quad \text{M1}$$

$$k = 1 - 0.65 = 0.35 \quad \text{A1}$$

Note: Always use sum-to-one as the very first step whenever a probability table contains an unknown; do not attempt to read off k by comparing individual values.

Q2 — Find k then $E(Y)$ EASY

(a) All probabilities must sum to 1, so we set up the equation.

$$0.4 + k + 0.25 = 1 \Rightarrow k = 0.35 \quad \text{A1}$$

(b) $E(Y)$ is the sum of each value multiplied by its probability.

$$E(Y) = 0(0.4) + 1(0.35) + 2(0.25) \quad \text{M1}$$

$$= 0 + 0.35 + 0.50 = 0.85 \quad \text{A1}$$

Note: Do not omit the 0×0.4 term; multiplying by zero contributes nothing, but forgetting to include all values in the product-sum is a common error.

Q3 — Expected Value from Given Probabilities EASY

$E(W)$ is calculated by multiplying each outcome by its probability and summing.

$$E(W) = 1(0.5) + 2(0.3) + 3(0.2) \quad \text{M1}$$

$$= 0.5 + 0.6 + 0.6 = 1.70 \quad \text{A1}$$

$$\text{We can verify the probabilities sum to 1: } 0.5 + 0.3 + 0.2 = 1 \quad \text{A1}$$

Note: $E(W) = 1.70$ does not mean the spinner can actually show 1.70; the expected value is the long-run average outcome, not a possible value of W .

Q4 — Find k and Read Off Probability EASY

(a) All probabilities sum to 1, giving a linear equation in k .

$$k + 0.3 + 2k + 0.1 = 1 \quad \text{M1}$$

$$3k = 0.6 \Rightarrow k = 0.2 \quad \text{A1}$$

(b) Substituting $k = 0.2$ into the expression for $P(X = 3) = 2k$.

$$P(X = 3) = 2(0.2) = 0.4 \quad \text{A1}$$

Note: $P(X = 3) = 2k$, not k ; always substitute back into the original expression rather than simply

writing k .

Q5 — Show Sum and Find $E(X)$ from Formula EASY

(a) We list each probability and add them to confirm the sum equals 1.

$$P(X=1) + P(X=2) + P(X=3) + P(X=4) = \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1 \quad \text{AG}$$

(b) We apply the expected value formula $E(X) = \sum x \cdot P(X = x)$.

$$E(X) = 1\left(\frac{1}{10}\right) + 2\left(\frac{2}{10}\right) + 3\left(\frac{3}{10}\right) + 4\left(\frac{4}{10}\right) \quad \text{M1}$$

$$= \frac{1 + 4 + 9 + 16}{10} = \frac{30}{10} = 3.00 \quad \text{A1}$$

Note: The formula $P(X = x) = x/10$ defines the probability for each value of x ; do not confuse this formula with the value of x itself when computing the product $x \cdot P(X = x)$.

Q6 — Expected Value and Tail Probability EASY

(a) We apply $E(X) = \sum x \cdot P(X = x)$ across all five values.

$$E(X) = 0(0.05) + 1(0.20) + 2(0.40) + 3(0.25) + 4(0.10) \quad \text{M1}$$

$$= 0 + 0.20 + 0.80 + 0.75 + 0.40 = 2.15 \quad \text{A1}$$

(b) $P(X \geq 3)$ is the sum of the probabilities for $x = 3$ and $x = 4$.

$$P(X \geq 3) = 0.25 + 0.10 = 0.35 \quad \text{A1}$$

Note: $P(X \geq 3)$ includes $x = 3$; do not write $P(X > 3) = 0.10$ by mistake.

Q7 — Expected Prize on a Single Spin EASY

Let X be the prize in dollars. We apply $E(X) = \sum x \cdot P(X = x)$.

$$E(X) = 0(0.60) + 5(0.30) + 10(0.10) \quad \text{M1}$$

$$= 0 + 1.50 + 1.00 = 2.50 \quad \text{A1}$$

The expected prize per spin is \$2.50. A1

Note: The 0×0.60 term is zero and does not affect the sum, but it must be included in the written expression to show a complete method.

Q8 — Find k then $E(T)$ EASY

(a) Because probabilities sum to 1, we solve for k .

$$0.3 + k + 0.2 = 1 \Rightarrow k = 0.5 \quad \text{A1}$$

(b) We apply $E(T) = \sum t \cdot P(T = t)$ using $k = 0.5$.

$$E(T) = 2(0.3) + 4(0.5) + 6(0.2)$$

$$= 0.6 + 2.0 + 1.2 = 3.80$$

M1

A1

Note: Always find k first and substitute it before computing $E(T)$; using an unknown k inside the product-sum is a common source of error.

Q9 — Expected Score on a Biased Die **EASY**

We apply $E(X) = \sum x \cdot P(X = x)$ using all four given probabilities.

$$E(X) = 1(0.1) + 2(0.4) + 3(0.3) + 4(0.2)$$

M1

$$= 0.1 + 0.8 + 0.9 + 0.8 = 2.60$$

A1

We verify the probabilities sum to 1: $0.1 + 0.4 + 0.3 + 0.2 = 1.0$

A1

Note: Check that the four probabilities sum to 1 before computing $E(X)$; if they do not, there is an error in the question or your reading of it.

Q10 — Find k from Formula then $E(M)$ **EASY**

(a) The sum of all probabilities equals 1, so we use $m = 1, 2, 4$.

$$\frac{k}{1} + \frac{k}{2} + \frac{k}{4} = 1$$

M1

$$k(1 + 0.5 + 0.25) = 1 \Rightarrow 1.75k = 1 \Rightarrow k = \frac{4}{7} \approx 0.571$$

A1

(b) We apply $E(M) = \sum m \cdot P(M = m)$.

$$E(M) = 1 \cdot \frac{4/7}{1} + 2 \cdot \frac{4/7}{2} + 4 \cdot \frac{4/7}{4} = \frac{4}{7} + \frac{4}{7} + \frac{4}{7} = \frac{12}{7} \approx 1.71$$

A1

Note: Each probability is k/m , not $k \cdot m$; substitute $k = 4/7$ carefully into each term before multiplying by m .

Q11 — Carnival Wheel: Find k and $E(X)$ **MEDIUM**

(a) Because all probabilities must sum to 1, we set up the equation $0.40 + k + 2k + 0.10 = 1$.

M1

$$\text{Solving: } 0.50 + 3k = 1 \Rightarrow 3k = 0.50 \Rightarrow k = \frac{1}{6} \approx 0.167.$$

A1

(b) Using $E(X) = \sum x \cdot P(X = x)$, multiply each token value by its probability and sum:

$$E(X) = 0(0.40) + 1\left(\frac{1}{6}\right) + 3\left(\frac{2}{6}\right) + 5(0.10) = 0 + \frac{1}{6} + 1 + 0.5$$

M1

$$\text{Unrounded: } \frac{1}{6} + 1.5 = \frac{1+9}{6} = \frac{10}{6} = 1.6\bar{6}; E(X) = 1.67 \text{ tokens (3 s.f.)}$$

A1

Note: A common error is to omit the factor of 2 in $2k$ when forming the sum-to-1 equation, giving an incorrect $k = 0.25$. Always write out every term before collecting.

Q12 — Café Loyalty Card: Two-Condition Parameter Hunt MEDIUM

Since all probabilities must sum to 1, the first equation is $a + 0.35 + b + 0.05 = 1$, giving $a + b = 0.60$.
M1

The expected value condition gives the second equation: $0(a) + 2(0.35) + 5b + 10(0.05) = 1.65$, so
 $0.70 + 5b + 0.50 = 1.65 \Rightarrow 5b = 0.45 \Rightarrow b = 0.09$. **A1**

Substituting back: $a = 0.60 - 0.09 = 0.51$. **A1**

GDC check: Equation solver (or simultaneous equations): enter $a + b = 0.60$ and $5b = 0.45$; confirms
 $a = 0.51, b = 0.09$. **A1**

Note: The term $0 \times a = 0$ contributes nothing to $E(R)$ but a still appears in the sum-to-1 equation — do not ignore a when solving the system.

Q13 — Charity Raffle: Expected Net Winnings and Fair Game MEDIUM

(a) The ticket costs \$3. Possible net winnings: winning \$150 gives $W = 147$; winning \$50 gives $W = 47$; winning \$10 gives $W = 7$; winning nothing gives $W = -3$. **A1**

(b) With 200 tickets: $P(W=147) = \frac{1}{200}$, $P(W=47) = \frac{2}{200}$, $P(W=7) = \frac{5}{200}$, $P(W=-3) = \frac{192}{200}$. **M1**

$E(W) = 147 \cdot \frac{1}{200} + 47 \cdot \frac{2}{200} + 7 \cdot \frac{5}{200} + (-3) \cdot \frac{192}{200} = \frac{147 + 94 + 35 - 576}{200} = \frac{-300}{200} = -1.50$. **A1**

(c) Since $E(W) = -1.50 < 0$, on average a player loses money per ticket. **R1**
Therefore buying a ticket is not a fair game.

Note: Net winnings must subtract the ticket cost of \$3 from every prize value — forgetting this gives $E(W) = 1.50$ (positive), reversing the conclusion.

Q14 — Formula Distribution: Show k and Find $E(X)$ MEDIUM

(a) Evaluate $k(x^2 + 1)$ for each x : $x = 0 : k(1)$; $x = 1 : k(2)$; $x = 2 : k(5)$; $x = 3 : k(10)$. The probabilities must sum to 1: **M1**

$k(1 + 2 + 5 + 10) = 1 \Rightarrow 18k = 1 \Rightarrow k = \frac{1}{18}$. **AG**

(b) Using $E(X) = \sum x \cdot P(X = x)$ with $k = \frac{1}{18}$: **M1**

$E(X) = 0 \cdot \frac{1}{18} + 1 \cdot \frac{2}{18} + 2 \cdot \frac{5}{18} + 3 \cdot \frac{10}{18} = \frac{0 + 2 + 10 + 30}{18} = \frac{42}{18} = \frac{7}{3} = 2.33$ (3 s.f.). **A1**

Note: In part (a) the target $k = 1/18$ is an AG — show the full sum $1 + 2 + 5 + 10 = 18$ explicitly; no marks are awarded for simply stating the answer.

Q15 — Minimum of Two Dice: Complete Table and $E(M)$ MEDIUM

(a) There are $4 \times 4 = 16$ equally likely outcomes. $P(M = m) = \frac{2(4 - m) + 1}{16} = \frac{9 - 2m}{16}$. **M1**

For $m = 2$: $\frac{5}{16}$; for $m = 3$: $\frac{3}{16}$. (Check: $\frac{7 + 5 + 3 + 1}{16} = \frac{16}{16} = 1$.) **A1**

(b) $E(M) = 1 \cdot \frac{7}{16} + 2 \cdot \frac{5}{16} + 3 \cdot \frac{3}{16} + 4 \cdot \frac{1}{16}$ **M1**
 $= \frac{7 + 10 + 9 + 4}{16} = \frac{30}{16} = 1.875 = 1.88$ (3 s.f.). **A1**

Note: The number of outcomes where the minimum equals m is $2(4 - m) + 1$, not simply $(4 - m)$ — list all pairs systematically (e.g. for $m = 2$: outcomes $(2, 2), (2, 3), (2, 4), (3, 2), (4, 2)$ give 5 outcomes) to avoid undercounting.

Q16 — Die Game: Expected Net Gain per Play and over 30 Plays MEDIUM

(a) The possible net gains are: roll a 6 $\Rightarrow G = 15 - 4 = 11$; roll 4 or 5 $\Rightarrow G = 5 - 4 = 1$; roll 1, 2, or 3 $\Rightarrow G = 0 - 4 = -4$. **M1**

$P(G=11) = \frac{1}{6}, P(G=1) = \frac{2}{6}, P(G=-4) = \frac{3}{6}$.
 $E(G) = 11 \cdot \frac{1}{6} + 1 \cdot \frac{2}{6} + (-4) \cdot \frac{3}{6} = \frac{11 + 2 - 12}{6} = \frac{1}{6} = 0.1\bar{6}$; $E(G) = \$0.167$ (3 s.f.). **A1**

Note: Net gain requires subtracting the \$4 entry fee from each prize; using gross prize values (\$15, \$5, \$0) without subtracting the fee is the primary trap here.

(b) By linearity of expectation, the expected total net gain over 30 plays is $30 \times \frac{1}{6} = 5$; expected gain = \$5.00. **A1**

Note: The per-play expected gain (\$0.167) must be multiplied by 30 — do not round $E(G)$ to \$0.17 before multiplying, as $30 \times 0.17 = \$5.10 \neq \5.00 .

Q17 — Goals Scored: Find k and $E(X)$; Contextual Interpretation MEDIUM

(a) All probabilities must sum to 1: $0.15 + 0.30 + k + 0.15 + 0.05 = 1 \Rightarrow 0.65 + k = 1 \Rightarrow k = 0.35$. **A1**

(b) $E(X) = 0(0.15) + 1(0.30) + 2(0.35) + 3(0.15) + 4(0.05)$ **M1**
 $= 0 + 0.30 + 0.70 + 0.45 + 0.20 = 1.65$ goals. **A1**

(c) On average, the school football team is expected to score 1.65 goals per match over many matches. **A1**

Note: The value $E(X) = 1.65$ does not mean the team will score 1.65 goals in any single match; it is a long-run average. Stating “the team will score 1.65 goals” without the long-run qualifier is an incomplete interpretation.

Q18 — Spinner with Proportional Probabilities: Show c ; $P(X \geq 3)$; $E(X)$ MEDIUM

(a) Since $P(X = x) = cx$ and the sum equals 1: $c(1 + 2 + 3 + 4) = 1 \Rightarrow 10c = 1 \Rightarrow c = 0.1$. **AG**

(b) $P(X \geq 3) = P(X=3) + P(X=4) = 0.1(3) + 0.1(4) = 0.3 + 0.4 = 0.7$. **A1**

(c) $E(X) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.4)$ **M1**
 $= 0.1 + 0.4 + 0.9 + 1.6 = 3.0$. **A1**

Note: In part (a) the target $c = 0.1$ is an AG — the line $10c = 1$ must be shown explicitly; writing only $c = 0.1$ earns no marks.

Q19 — Counters Without Replacement: Distribution and $E(Y)$ MEDIUM

(a) Drawing without replacement from 4 red and 6 blue (10 total). **M1**

$P(Y=0) = \frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$; $P(Y=1) = \frac{4}{10} \cdot \frac{6}{9} + \frac{6}{10} \cdot \frac{4}{9} = \frac{48}{90} = \frac{8}{15}$; $P(Y=2) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$. **A1**

(b) $E(Y) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{8}{15} + 2 \cdot \frac{2}{15} = 0 + \frac{8}{15} + \frac{4}{15} = \frac{12}{15} = \frac{4}{5} = 0.8$. **M1A1**

Note: This is a without-replacement scenario — the denominator changes from 10 to 9 on the second draw. Using $\frac{4}{10} \cdot \frac{4}{10}$ (with-replacement denominators) is incorrect and is a classic trap in this topic.

Q20 — Two-Condition Parameter Hunt: Find p and q MEDIUM

Since all probabilities must sum to 1, the first equation is $p + 0.20 + q + 0.10 = 1$, giving: **M1**

$p + q = 0.70$. **A1**

The expected value condition gives: $1(p) + 2(0.20) + 4q + 8(0.10) = 3.2 \Rightarrow p + 4q + 0.40 + 0.80 = 3.2 \Rightarrow p + 4q = 2.00$. **M1**

Subtracting the first equation from the second: $3q = 1.30 \Rightarrow q = \frac{13}{30} \approx 0.433$; then $p = 0.70 - 0.433 = \frac{4}{15} \approx 0.267$.

GDC: Equation solver $\rightarrow$ enter $p+q = 0.70$ and $p+4q = 2.00$; solution $p = \frac{4}{15} \approx 0.267$, $q = \frac{13}{30} \approx 0.433$. **A1**

Note: The value $x = 8$ with coefficient 0.10 contributes $8 \times 0.10 = 0.80$ to $E(X)$, not 0.10 — forgetting to multiply the x -value by its probability is the trap that leads to an incorrect second equation.

Q21 — Spinner Score Distribution and Fair Game Decision HARD

(a) Since all probabilities must sum to 1, we write the sum-to-1 equation as the essential first step.

$2k + 3k + (k + 0.1) + 0.3 = 1$ **M1**

$$6k + 0.4 = 1 \Rightarrow 6k = 0.6 \Rightarrow k = 0.1.$$

A1 AG

Note: Collect all four terms carefully — $2k + 3k + (k + 0.1) + 0.3 = 6k + 0.4$; a common slip is to mis-add the constants and obtain an incorrect coefficient.

(b) Using $k = 0.1$, the probabilities are $P(1) = 0.2, P(2) = 0.3, P(3) = 0.2, P(4) = 0.3$.

$$E(X) = 1(0.2) + 2(0.3) + 3(0.2) + 4(0.3) = 0.2 + 0.6 + 0.6 + 1.2$$

M1

$$= 2.60.$$

A1

(c) The expected winnings per play equal $E(X) - \text{fee} = 2.60 - 1.50 = 1.10 > 0$, so the game favours the player.

Since $E(X) = 2.60 \neq 1.50$, the game is not fair; the player has a positive expected gain of \$1.10 per spin.

R1

Note: A fair game requires $E(\text{net gain}) = 0$, i.e. $\text{fee} = E(X)$; do not confuse “fair” with “the player wins on average.”

Q22 — Formula-Defined Distribution and $E(Y)$ **HARD**

(a) We first compute $P(Y = y)$ for each value, then use sum-to-1 to find c .

$$P(Y=0) = c(1), P(Y=1) = c(2), P(Y=2) = c(5), P(Y=3) = c(10).$$

$$\text{Sum: } c(1 + 2 + 5 + 10) = 1 \Rightarrow 18c = 1$$

M1

$$c = \frac{1}{18} \approx 0.0556$$

A1

$$\text{(b)} P(Y \geq 2) = P(Y=2) + P(Y=3) = \frac{5}{18} + \frac{10}{18} = \frac{15}{18} = \frac{5}{6} \approx 0.833$$

A1

(c) Using the definition $E(Y) = \sum y \cdot P(Y = y)$:

$$E(Y) = 0 \cdot \frac{1}{18} + 1 \cdot \frac{2}{18} + 2 \cdot \frac{5}{18} + 3 \cdot \frac{10}{18} = 0 + \frac{2}{18} + \frac{10}{18} + \frac{30}{18}$$

M1

$$= \frac{42}{18} = \frac{7}{3} \approx 2.33$$

A1

Note: Do not forget the $y = 0$ term; although it contributes zero to $E(Y)$, omitting $P(Y = 0)$ from the sum-to-1 step gives an incorrect value of c .

Q23 — Without-Replacement Token Draws and $E(W)$ **HARD**

(a) Drawing 2 tokens without replacement from 3 red and 2 blue (5 total); we count ordered pairs.

$$P(W=2) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

M1

$$= 0.3$$

A1 AG

Note: Using with-replacement gives $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$, which destroys the AG.

(b) The possible values of W are 0, 1, 2. We find the remaining probabilities by counting.

$$P(W=0) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}; \quad P(W=1) = 1 - \frac{1}{10} - \frac{3}{10} = \frac{6}{10} = \frac{3}{5}$$

M1

$$(\text{Verify: } P(W=1) = \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5} \checkmark)$$

$$E(W) = 0 \cdot \frac{1}{10} + 1 \cdot \frac{6}{10} + 2 \cdot \frac{3}{10} = 0 + \frac{6}{10} + \frac{6}{10} = \frac{12}{10} = 1.2$$

A1

This equals $2 \times \frac{3}{5} = 1.2$, consistent with the hypergeometric mean.

A1

Note: The two-red-one-blue and one-red-two-blue cases each have two orderings (RB and BR); failing to count both orderings halves $P(W = 1)$ and invalidates $E(W)$.

Q24 — Minimum of Two Dice: Constructed Distribution **HARD**

(a) The minimum equals 1 unless both dice show a value ≥ 2 .

$$P(M=1) = 1 - P(\text{both} \geq 2) = 1 - \frac{5}{6} \times \frac{5}{6} = 1 - \frac{25}{36} = \frac{11}{36}$$

M1

$$\approx 0.306$$

A1 AG

Note: A common error is to count only the cases where exactly one die shows 1; the complement method is safer.

$$(b) P(M=6) \text{ requires both dice to show 6: } P(M=6) = \frac{1}{36}$$

A1

(c) Using the general formula $P(M \geq m) = \left(\frac{7-m}{6}\right)^2$, so $P(M = m) = P(M \geq m) - P(M \geq m+1)$:

$$P(M = m) = \frac{(7-m)^2 - (6-m)^2}{36} = \frac{2(6-m) + 1}{36} = \frac{13-2m}{36} \text{ for } m = 1, \dots, 6.$$

M1

GDC: Stat List, enter $x : 1, 2, 3, 4, 5, 6$ and $P : \frac{11}{36}, \frac{9}{36}, \frac{7}{36}, \frac{5}{36}, \frac{3}{36}, \frac{1}{36}$; compute $\sum x \cdot P$.

$$= \frac{11 + 18 + 21 + 20 + 15 + 6}{36} = \frac{91}{36} \approx 2.528 \approx 2.53$$

A1

Note: Using the maximum formula $(2m-1)/36$ instead of the minimum formula gives $E(M) = \frac{161}{36}$, an entirely different answer.

Q25 — Two-Condition Parameter Hunt via Simultaneous Equations **HARD**

Since all probabilities must sum to 1, the first equation comes from the sum-to-1 condition.

$$a + 0.3 + b + 0.1 = 1 \Rightarrow a + b = 0.6$$

M1

The second equation comes from the given expected value $E(X) = 1.2$:

$$0(a) + 1(0.3) + 2(b) + 3(0.1) = 1.2 \Rightarrow 0.3 + 2b + 0.3 = 1.2 \Rightarrow 2b = 0.6 \Rightarrow b = 0.3$$

M1

GDC: Equation solver or substitution: from $a + b = 0.6$ and $b = 0.3$, gives $a = 0.3$.

$$b = 0.3$$

A1

$$a = 0.3$$

A1

$$\text{Verify: } E(X) = 0(0.3) + 1(0.3) + 2(0.3) + 3(0.1) = 0 + 0.3 + 0.6 + 0.3 = 1.2 \checkmark$$

A1

Note: Setting up $E(X)$ with $x = 0, 1, 2, 3$ and mistakenly omitting the $x = 0$ term (which contributes $0 \cdot a = 0$) does not affect the $E(X)$ equation here, but failing to include a in the sum-to-1 equation gives an incorrect value of b .

Q26 — Fair Game Fee and Expected Profit over N Plays **HARD**

(a) We identify the prize structure: win \$5 with probability $\frac{1}{6}$, win \$2 with probability $\frac{2}{6}$, win \$0 with probability $\frac{3}{6}$.

$$E(W) = 5 \times \frac{1}{6} + 2 \times \frac{2}{6} + 0 \times \frac{3}{6} = \frac{5}{6} + \frac{4}{6} + 0 \quad \text{M1}$$

$$= \frac{9}{6} = \frac{3}{2} = 1.50 \quad \text{A1}$$

(b) For a fair game, the entry fee equals the expected winnings.

$$c = E(W) = \$1.50 \quad \text{A1}$$

(c) Expected profit per play = $E(W) - c = 1.50 - 1.50 = 0$. Over 30 plays:

$$\text{Expected total profit} = 30 \times 0 = \$0 \quad \text{M1}$$

The expected total profit over 30 plays is exactly \$0.00. A1

Note: A common trap is to compute $30 \times E(W) = \$45$ and forget to subtract the total entry fees $30c = \$45$; the net result is \$0, not \$45. Always use net gain, not gross winnings.

Q27 — Formula Distribution: Expected Loss and Rational Decision **HARD**

(a) Computing $P(X = x) = k(4 - x)$ for $x = 0, 1, 2, 3$: values are $4k, 3k, 2k, k$.

$$\text{Sum} = 4k + 3k + 2k + k = 10k = 1 \Rightarrow k = 0.1 \quad \text{A1}$$

(b) Probabilities: $P(0) = 0.4, P(1) = 0.3, P(2) = 0.2, P(3) = 0.1$.

$$E(X) = 0(0.4) + 1(0.3) + 2(0.2) + 3(0.1) = 0 + 0.3 + 0.4 + 0.3 \quad \text{M1}$$

$$= 1.0 \quad \text{A1}$$

(c) Expected net gain per game = $E(X) - \text{fee} = 1.00 - 0.80 = +\0.20 .

Since the expected net gain per game is $+\$0.20 > 0$, the player gains on average; a rational player should participate. R1

Expected loss per game = $-\$0.20$ (i.e. there is an expected gain of \$0.20). A1

Note: "Expected loss" is fee $- E(X)$; if $E(X) > \text{fee}$, the result is negative, meaning the player actually gains. Read "expected loss" carefully — a negative loss is a gain.

Q28 — Raffle Net Gain Distribution: $E(G)$ and Rational Decision **HARD**

(a) The ticket costs \$4. Net gains: first prize win gives $80 - 4 = \$76$; second prize win gives $20 - 4 = \$16$; no prize gives $0 - 4 = -\$4$.

Probabilities: $P(G=76) = \frac{1}{50}$; $P(G=16) = \frac{2}{50}$; $P(G=-4) = \frac{47}{50}$ **M1**

(Table correctly constructed with all three net values and probabilities summing to 1.) **A1**

(b) $E(G) = 76 \times \frac{1}{50} + 16 \times \frac{2}{50} + (-4) \times \frac{47}{50}$ **M1**

$$= \frac{76 + 32 - 188}{50} = \frac{-80}{50} = -1.60$$

$E(G) = -\$1.60$ **A1**

(c) Since $E(G) = -1.60 < 0$, the player expects to lose \$1.60 per ticket purchased; buying a ticket is not financially rational. **R1**

Note: A common error is to use the prize values (\$80, \$20, \$0) without subtracting the \$4 ticket cost; this gives $E(\text{prize}) = +\$2.40$ instead of $E(G) = -\$1.60$, reversing the conclusion.

Q29 — Negative-Value Distribution and Two-Condition Parameter Hunt **HARD**

The two conditions are sum-to-1 and the given $E(X) = 0$; these form two simultaneous equations.

Sum-to-1: $p + 0.25 + q + 0.05 = 1 \Rightarrow p + q = 0.70$ **M1**

Expected value: $(-1)(p) + 0(0.25) + 1(q) + 2(0.05) = 0$

$\Rightarrow -p + q + 0.10 = 0 \Rightarrow -p + q = -0.10$ **M1**

GDC: Equation solver (or by hand): Adding the two equations:

$(p + q) + (-p + q) = 0.70 + (-0.10) \Rightarrow 2q = 0.60 \Rightarrow q = 0.30$ **A1**

$p = 0.70 - 0.30 = 0.40$ **A1**

Verify $p > q$: $0.40 > 0.30$ ✓ **A1**

Note: The $x = -1$ term contributes $-p$ (negative) to $E(X)$; writing $+p$ instead gives $p + q = 0.10$, yielding negative probabilities and an invalid distribution.

Q30 — Defective Bulb Distribution: Scaled $E(D)$ and Acceptability Decision **HARD**

(a) Sum-to-1 gives: $0.5 + m + 0.1 + 0.02 = 1 \Rightarrow m = 0.38$ **A1**

(b) $E(D) = 0(0.5) + 1(0.38) + 2(0.1) + 3(0.02)$ **M1**

$= 0 + 0.38 + 0.20 + 0.06 = 0.64$ **A1**

(c) Over 5 independent boxes, the expected total number of defective bulbs is $5 \times E(D)$.

$5 \times 0.64 = 3.20$ **M1**

Since $3.20 \geq 2$, the expected total number of defective bulbs is not less than 2; the process is not acceptable. **R1**

Note: Comparing $E(D) = 0.64$ directly to 2 (instead of scaling by 5) incorrectly concludes the process is acceptable; the question asks about the total across 5 boxes, so the correct comparison is $5 \times 0.64 = 3.20$ vs. 2.