IB Math AI SL — Topic 4

Probability

Events · Independence & Mutual Exclusivity · Conditional Probability · Venn & Tree Diagrams

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Combined events:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Complement:** $P(A') = 1 - P(A)$
- **Conditional probability:** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$
- **Independent events test:** A and B are independent $\iff$ $P(A \cap B) = P(A) \times P(B)$
- **Mutually exclusive events test:** A and B are mutually exclusive $\iff$ $P(A \cap B) = 0$
- **At least one (complement method):** $P(\text{at least one}) = 1 - P(\text{none})$
- **Total probability from a tree (two stages):** $P(B) = P(A)\,P(B \mid A) + P(A')\,P(B \mid A')$

GDC Tips

- **Storing fractions exactly:** Enter probabilities as fractions (e.g. `3/7`) and press `EXE`; the GDC stores the exact decimal. Use `MATH` $\to$ `Frac` (TI) or `OPTN` $\to$ `CALC` $\to$ `b/c` (Casio) to convert a decimal back to a fraction and verify.
- **Verifying independence:** Store $P(A)$, $P(B)$, and $P(A \cap B)$ as variables `A`, `B`, `C`; compute `A`×`B` and compare with `C` directly on the home screen.
- **Two-way table counts:** Enter the contingency table into `MATRIX` (TI: `2nd` $\to$ `MATRIX` $\to$ `EDIT`) or `List` mode (Casio); use row/column sums to read off marginal totals quickly.
- **Checking Venn totals:** Sum all region values on the home screen; confirm they equal the universal set total $n(U)$ before extracting any probability.
- **Tree diagram multiplication:** Multiply branch probabilities along each path on the home screen (e.g. `(3/7)*(2/6)` for without-replacement); add the required path products for combined outcomes.
- **Without-replacement check:** After drawing the first branch, mentally reduce the denominator by 1 and the relevant numerator by 1; store the new fraction before computing to avoid keying errors.

Common Mistakes to Avoid

1. **Confusing independent and mutually exclusive.** Students often state that two events are independent because they "don't overlap". These are different properties: mutually exclusive means $P(A \cap B) = 0$ (they cannot both occur), while independent means $P(A \cap B) = P(A) \times P(B)$ (knowing one gives no information about the other). Always name the test you are applying and show the numerical check.

2. **Forgetting to reduce the denominator in without-replacement trees.** When items are selected without replacement, the second-stage denominator decreases by 1 (and the matching numerator may also decrease). Writing the same denominator on both stages is the single most common tree-diagram error. Label every branch with its own fraction after the previous selection.

3. **Reversing the conditional probability.** $P(A \mid B)$ and $P(B \mid A)$ are generally not equal. Always identify which event is the *given* condition (the denominator) and which is the *target* (the numerator) before substituting into $P(A \mid B) = P(A \cap B)/P(B)$.

4. **Double-counting in the addition rule.** Using $P(A \cup B) = P(A) + P(B)$ without subtracting $P(A \cap B)$ overcounts the intersection. The simplified form $P(A) + P(B)$ is only valid when the events are mutually exclusive (intersection is empty). Always check whether $P(A \cap B) = 0$ before omitting the subtraction term.

5. **Using the wrong total in a Venn diagram probability.** After solving for the unknown region x, students sometimes divide by the number of regions instead of the total number of outcomes $n(U)$. Sum all regions first, confirm the total equals $n(U)$, and use that total as every probability denominator.

6. **Premature rounding in chained probability calculations.** Rounding an intermediate branch probability (e.g. to 2 decimal places) before multiplying along the tree introduces cumulative error. Keep exact fractions or full calculator precision throughout, and round only the final answer to the required accuracy (3 significant figures unless stated otherwise).

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Section A — Easy

EASY

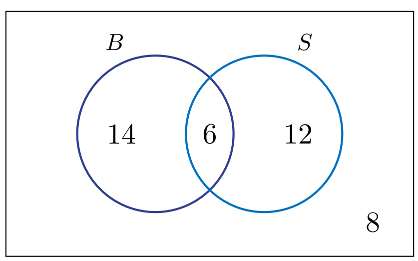
1. A bag contains 5 red marbles and 3 blue marbles. One marble is chosen at random.

(a) Write down the probability that the marble chosen is red. **[1 marks]**

(b) Write down the probability that the marble chosen is not red. **[1 marks]**

(c) Write down the probability that the marble chosen is green. **[1 marks]**

2. A survey of 40 students asked whether they own a bicycle (B) or a skateboard (S). The Venn diagram below shows the results.



One student is selected at random.

(a) Write down the number of students who own both a bicycle and a skateboard. **[1 marks]**

(b) Find the probability that the student owns a bicycle. **[1 marks]**

(c) Find the probability that the student owns neither a bicycle nor a skateboard. **[1 marks]**

3. Events A and B are such that $P(A) = 0.4$, $P(B) = 0.3$, and A and B are mutually exclusive.

(a) Write down $P(A \cap B)$. **[1 marks]**

(b) Find $P(A \cup B)$. **[1 marks]**

(c) Find $P(A')$. **[1 marks]**

4. Events C and D are independent with $P(C) = 0.5$ and $P(D) = 0.4$.

(a) Find $P(C \cap D)$. **[1 marks]**

(b) Find $P(C \cup D)$. **[1 marks]**

(c) Write down whether C and D can also be mutually exclusive. Give a reason. **[1 marks]**

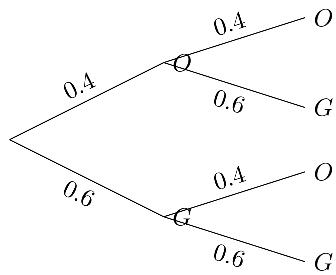
5. A spinner has five equal sectors numbered 1 to 5. The spinner is spun once.

(a) Find the probability that the spinner lands on an even number. **[1 marks]**

(b) Find the probability that the spinner lands on a number greater than 3. **[1 marks]**

(c) Find the probability that the spinner lands on an even number **or** a number greater than 3. **[1 marks]**

6. A box contains 4 orange sweets and 6 green sweets. One sweet is chosen at random, its colour noted, and it is replaced. A second sweet is then chosen at random.
The tree diagram below shows the possible outcomes.



(a) Write down the probability of choosing two orange sweets. **[1 marks]**

(b) Find the probability of choosing one sweet of each colour. **[2 marks]**

7. In a class of 30 students, 18 study French (F) and 12 study Spanish (S). No student studies both languages.

(a) Write down $P(F \cap S)$. **[1 marks]**

(b) Find the probability that a randomly chosen student studies French or Spanish. **[1 marks]**

(c) Determine whether F and S are mutually exclusive. Justify your answer. **[1 marks]**

8. The probability that Mia catches the early bus on any morning is 0.7. The probability that she arrives at school on time, given she catches the early bus, is 0.9. The probability that she arrives on time, given she does **not** catch the early bus, is 0.3.
Find the probability that Mia catches the early bus **and** arrives at school on time. **[3 marks]**

9. A class has 25 students. Each student studies exactly one of three subjects: Art (A), Biology (B), or Chemistry (C). The table below shows the number of students studying each subject, by gender.

	Art	Biology	Chemistry	Total
Female	5	7	3	15
Male	4	3	3	10
Total	9	10	6	25

One student is chosen at random.

(a) Write down the probability that the student studies Biology. **[1 marks]**

(b) Find the probability that the student is female and studies Art. **[1 marks]**

(c) Find the probability that the student studies Chemistry, given that the student is male. **[1 marks]**

10. Events P and Q are independent with $P(P) = 0.3$ and $P(Q) = 0.6$.

(a) Find $P(P \cap Q)$. **[1 marks]**

(b) Find $P(P \cup Q)$. **[1 marks]**

(c) Find $P(P' \cap Q')$. **[1 marks]**

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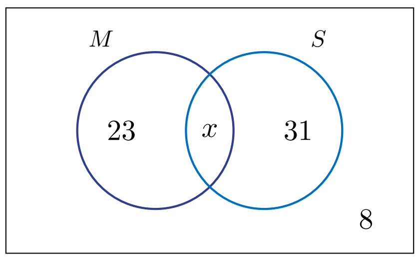
Section B — Medium

MEDIUM

11. A survey of 80 students asked whether they study Mathematics (M) or Science (S). The results are shown in the Venn diagram below.



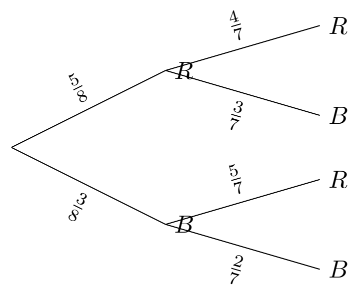
(a) Find the value of x. **[1 marks]**

(b) Find the probability that a randomly selected student studies Mathematics but not Science. **[1 marks]**

(c) Find the probability that a randomly selected student studies at least one of the two subjects. **[1 marks]**

(d) Given that a student studies Science, find the probability that they also study Mathematics. **[1 marks]**

12. A bag contains 5 red counters and 3 blue counters. Two counters are selected at random, one after the other, **without replacement**. A tree diagram representing this situation is shown below.



(a) Find the probability that both counters selected are red. **[1 marks]**

(b) Find the probability that the two counters selected are of different colours. **[2 marks]**

(c) Find the probability that the second counter is blue. **[1 marks]**

13. Events A and B are such that $P(A) = 0.45$, $P(B) = 0.30$, and $P(A \cup B) = 0.62$.

(a) Find $P(A \cap B)$. **[2 marks]**

(b) Determine whether events A and B are independent. Justify your answer. **[2 marks]**

14. At a school canteen, 60% of students buy a hot meal (H) and 35% buy a cold drink (C). The probability that a student buys both a hot meal and a cold drink is 0.18.

(a) Find the probability that a randomly selected student buys a hot meal or a cold drink or both. **[1 marks]**

(b) Find the probability that a student buys a cold drink but not a hot meal. **[1 marks]**

(c) Determine whether buying a hot meal and buying a cold drink are independent events. Justify your answer. **[2 marks]**

15. A class of 30 students was asked about their after-school activities. The two-way table below shows the number of students who play sport (S) and those who attend art club (A).

	Art club (A)	**No art club**	**Total**
Sport (S)	7	11	18
No sport	6	6	12
Total	13	17	30

(a) Write down $P(S \cap A)$. **[1 marks]**

(b) Find the probability that a student plays sport given that they attend art club. **[2 marks]**

(c) Determine whether events S and A are mutually exclusive. Justify your answer. **[1 marks]**

16. A factory produces light bulbs. The probability that a randomly chosen bulb is defective is 0.04. Two bulbs are chosen at random, independently of each other.

(a) Find the probability that both bulbs are defective. **[1 marks]**

(b) Find the probability that exactly one bulb is defective. **[2 marks]**

(c) Find the probability that at least one bulb is defective. **[1 marks]**

17. In a group of 50 tourists, 28 visited Paris (P), 22 visited Rome (R), and 10 visited both cities. A tourist is selected at random.

(a) Draw a Venn diagram to represent this information, labelling all regions clearly. **[2 marks]**

(b) Find the probability that the tourist visited Paris or Rome, but not both. **[1 marks]**

(c) Given that the tourist visited Rome, find the probability that they also visited Paris. **[1 marks]**

18. A probability distribution for the discrete random variable X is given in the table below.

x	1	2	3	4
$P(X = x)$	0.15	k	0.35	$2k$

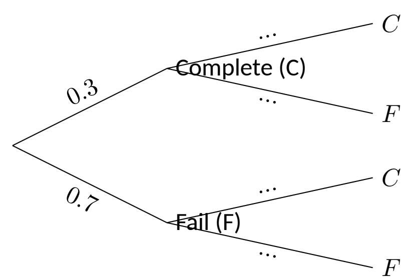
(a) Find the value of k. **[2 marks]**

(b) Find the probability that X is at most 3. **[1 marks]**

(c) Find $P(X = 2 \mid X \leq 3)$. **[1 marks]**

19. A student plays a video game. On each attempt, the probability of completing the level is 0.3, independently of other attempts. The student makes two attempts.

(a) Complete the tree diagram below by writing the correct probabilities on each branch. **[1 marks]**



(b) Find the probability that the student completes the level at least once. **[2 marks]**

(c) Given that the student completes the level at least once, find the probability that they completed it on both attempts. **[1 marks]**

20. Events A and B are independent with $P(A) = p$ and $P(B) = 0.4$. It is known that $P(A \cup B) = 0.70$.

(a) Show that p satisfies the equation $0.6p = 0.30$. **[2 marks]**

(b) Hence find the value of p. **[1 marks]**

(c) Find $P(A' \cap B)$, where A' denotes the complement of A. **[1 marks]**

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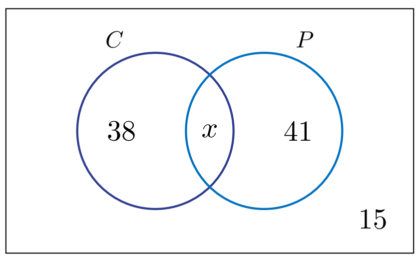
Section C — Hard

HARD

21. A survey of 120 students at an international school recorded whether each student plays chess (C), solves puzzles (P), or both. The results are shown in the Venn diagram below, where x represents the number of students who do both activities.



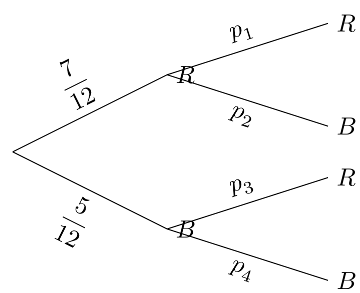
(a) Find the value of x. **[1 marks]**

(b) A student is selected at random. Find the probability that the student plays chess given that they solve puzzles. **[2 marks]**

(c) Determine whether events C and P are independent. Justify your answer. **[2 marks]**

22. A bag contains 7 red tokens and 5 blue tokens. Two tokens are drawn at random **without replacement**.

(a) Complete the tree diagram below to show all branch probabilities. **[2 marks]**



(b) Find the probability that the two tokens drawn are different colours. **[2 marks]**

(c) Find the probability that at least one token is red. **[1 marks]**

23. At a school sports day, the probability that a student wins their running race is 0.45. The probability that a student wins their jumping event is 0.30. The probability that a student wins **at least one** of these two events is 0.62.

(a) Show that the probability that a student wins **both** events is 0.13. **[2 marks]**

(b) Hence determine whether winning the running race and winning the jumping event are independent events. Justify your answer. **[2 marks]**

(c) Find the probability that a student wins the jumping event **given** that they win the running race. **[1 marks]**

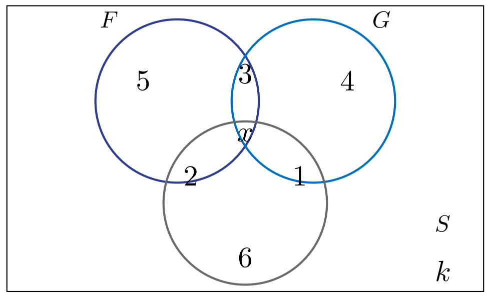
24. In a city, 60% of residents own a car and 35% own a bicycle. It is known that 20% of residents own both a car and a bicycle. A resident is selected at random.

(a) Find the probability that the resident owns a car or a bicycle (or both). **[1 marks]**

(b) Find the probability that the resident owns a bicycle given that they own a car. **[2 marks]**

(c) Two residents are selected at random. Find the probability that exactly one of them owns both a car and a bicycle. **[2 marks]**

25. A three-set Venn diagram shows the number of students in a class of 40 who study French (F), German (G), and Spanish (S). The diagram shows the following information: 5 study only French, 4 study only German, 6 study only Spanish, 3 study French and German only, 2 study French and Spanish only, 1 studies German and Spanish only, and x students study all three languages.



(a) Find the value of x and the value of k (students studying none of the three languages). **[2 marks]**

(b) A student is chosen at random. Find the probability that the student studies exactly two languages. **[1 marks]**

(c) Find the probability that a student studies French given that they study German. **[2 marks]**

26. The probability that a randomly selected flight from an airport departs on time is p. The probability that a flight both departs on time **and** arrives on time is 0.56. The probability that a flight arrives on time **given** that it departs on time is 0.80.

(a) Find the value of p. **[2 marks]**

(b) The probability that a flight arrives on time is q. Given that departure and arrival on time are independent events, find the value of q. **[2 marks]**

(c) Determine whether departure on time and arrival on time are in fact independent. Justify your answer. **[1 marks]**

27. A factory produces electronic components. The probability that a component is defective is 0.04. Components are tested one at a time. Let X be the number of defective components found in a batch of 3 components tested.

(a) Show that the probability of finding exactly one defective component in the batch is 0.111 (to 3 s.f.). **[2 marks]**

(b) Find the probability that at least two components in the batch are defective. **[2 marks]**

(c) Find the expected number of defective components in the batch. **[1 marks]**

28. Events A and B are such that $P(A) = 0.5$, $P(B) = k$, and $P(A \cup B) = 0.8$.

(a) Given that A and B are mutually exclusive, find the value of k. **[1 marks]**

(b) Given instead that A and B are independent, find the value of k. **[3 marks]**

(c) State one reason why A and B cannot be both mutually exclusive and independent (assuming $P(A) > 0$ and $P(B) > 0$). **[1 marks]**

29. A game at a school fair uses a spinner with three equal sectors coloured Red (R), Yellow (Y), and Blue (B). A player spins the spinner twice.

(a) Write down the probability of obtaining Red on a single spin. **[1 marks]**

(b) Find the probability that the two spins show the same colour. **[2 marks]**

(c) A player wins a prize if at least one spin shows Blue. Find the probability that a player does **not** win a prize. **[2 marks]**

30. In a group of 80 athletes, some compete in swimming (W) and some compete in cycling (Y). The two-way table below shows partial information about the athletes.

	Swims	Does not swim	Total
Cycles	a	12	
Does not cycle	18	b	
Total	38		80

(a) Find the values of a and b. **[2 marks]**

(b) Find the probability that a randomly selected athlete swims given that they cycle. **[1 marks]**

(c) Determine whether swimming and cycling are independent events for this group. Justify your answer. **[2 marks]**

Worked Solutions

Q1 — Basic probability from equally likely outcomes [EASY]

(a) The total number of marbles is $5+3=8$, so the probability of red is the number of red marbles divided by the total. **A1**

$$P(\text{red}) = \frac{5}{8}$$

(b) The event "not red" is the complement of red, so its probability is 1 minus the probability of red. **A1**

$$P(\text{not red}) = 1 - \frac{5}{8} = \frac{3}{8}$$

(c) There are no green marbles in the bag, so the event is impossible. **A1**

$$P(\text{green}) = 0$$

Q2 — Venn diagram probabilities [EASY]

(a) The intersection region of the two circles shows the number of students owning both items. Reading directly from the Venn diagram: **A1**

$$n(B \cap S) = 6$$

(b) The number of students who own a bicycle is the sum of the two regions inside circle B: $14+6=20$. Dividing by the total of 40 gives the probability. **A1**

$$P(B) = \frac{20}{40} = \frac{1}{2}$$

(c) The number of students outside both circles is 8 (the region in the rectangle but outside both circles). Dividing by 40: **A1**

$$P(\text{neither}) = \frac{8}{40} = \frac{1}{5}$$

Q3 — Mutually exclusive events [EASY]

(a) Mutually exclusive means the events cannot both occur, so their intersection is empty. **A1**

$$P(A \cap B) = 0$$

(b) For mutually exclusive events, the addition rule simplifies: $P(A \cup B) = P(A) + P(B)$. **A1**

$$P(A \cup B) = 0.4 + 0.3 = 0.7$$

(c) The complement of A is everything that is not A, so $P(A') = 1 - P(A)$. **A1**

$$P(A') = 1 - 0.4 = 0.6$$

Q4 — Independent events and mutual exclusivity [EASY]

(a) For independent events, the multiplication rule gives $P(C \cap D) = P(C) \times P(D)$. **A1**

$$P(C \cap D) = 0.5 \times 0.4 = 0.2$$

(b) Using the general addition rule $P(C \cup D) = P(C) + P(D) - P(C \cap D)$: **A1**

$$P(C \cup D) = 0.5 + 0.4 - 0.2 = 0.7$$

(c) Mutually exclusive would require $P(C \cap D) = 0$, but $P(C \cap D) = 0.2 \neq 0$, so C and D cannot be mutually exclusive. **A1**

Warning: Independent events are almost never mutually exclusive (unless one has probability 0); do not confuse the two concepts.

Q5 — Combined events on a spinner [EASY]

(a) The even numbers from 1 to 5 are $\{2, 4\}$, so 2 out of 5 sectors are even. **A1**

$$P(\text{even}) = \frac{2}{5}$$

(b) Numbers greater than 3 from 1 to 5 are $\{4, 5\}$, so 2 out of 5 sectors qualify. **A1**

$$P(\text{greater than } 3) = \frac{2}{5}$$

(c) The event "even **or** greater than 3" includes $\{2, 4, 5\}$, since 4 is in both sets (it is counted once). Using the addition rule: $P(E) + P(G) - P(E \cap G) = \frac{2}{5} + \frac{2}{5} - \frac{1}{5}$, where $E \cap G = \{4\}$. **A1**

$$P(\text{even or greater than } 3) = \frac{3}{5}$$

Q6 — Tree diagram with replacement [EASY]

(a) Because the sweet is replaced, all second-stage probabilities equal the first-stage probabilities. Reading the top-top branch of the tree: **A1**

$$P(OO) = 0.4 \times 0.4 = 0.16$$

(b) "One of each colour" means either Orange then Green (OG) or Green then Orange (GO). Multiply along each branch then add the two paths. **M1**

$$P(OG) + P(GO) = (0.4 \times 0.6) + (0.6 \times 0.4) = 0.24 + 0.24$$

$$= 0.48\textbf{A1}$$

Warning: With replacement, the second-stage denominator does NOT change; if the problem were without replacement the probabilities on the second set of branches would differ from the first.

Q7 — Mutually exclusive language classes [EASY]

(a) No student studies both languages, so the intersection contains no students. **A1**

$$P(F \cap S) = 0$$

(b) Since the events are mutually exclusive, $P(F \cup S) = P(F) + P(S)$. **A1**

$$P(F \cup S) = \frac{18}{30} + \frac{12}{30} = \frac{30}{30} = 1$$

(c) Testing mutual exclusivity: $P(F \cap S) = 0$. Since no student studies both, the events are mutually exclusive. **A1**
F and S are mutually exclusive because $P(F \cap S) = 0$.

Q8 — Tree diagram conditional probability [EASY]

To find $P(\text{early bus} \cap \text{on time})$, multiply the probability of catching the early bus by the conditional probability of arriving on time given the early bus was caught. **M1**

$$P(\text{early bus} \cap \text{on time}) = P(\text{early bus}) \times P(\text{on time} \mid \text{early bus})$$

$$= 0.7 \times 0.9\textbf{(M1)}$$

$$= 0.63\textbf{A1}$$

Warning: The question asks only for the single branch probability (early bus AND on time), not for the total probability of arriving on time from all routes; do not add any other branch.

Q9 — Two-way table conditional probability [EASY]

(a) The total number of Biology students is 10 out of 25. **A1**

$$P(B) = \frac{10}{25} = \frac{2}{5}$$

(b) Reading the Female–Art cell from the table: 5 students out of 25. **A1**

$$P(\text{female and Art}) = \frac{5}{25} = \frac{1}{5}$$

(c) For a conditional probability, restrict to the male row (total 10 males) and count those studying Chemistry (3 males). **A1**

$$P(\text{Chemistry} \mid \text{male}) = \frac{3}{10}$$

Warning: The denominator for the conditional probability is the total number of males (10), not the overall class total (25).

Q10 — Independent events complement [EASY]

(a) For independent events, multiply the individual probabilities. **A1**

$$P(P \cap Q) = 0.3 \times 0.6 = 0.18$$

(b) Using the general addition rule $P(P \cup Q) = P(P) + P(Q) - P(P \cap Q)$: **A1**

$$P(P \cup Q) = 0.3 + 0.6 - 0.18 = 0.72$$

(c) The event $P' \cap Q'$ is the complement of $P \cup Q$, so its probability is $1 - P(P \cup Q)$. **A1**

$$P(P' \cap Q') = 1 - 0.72 = 0.28$$

Q11 — Venn diagram with two sets [MEDIUM]

(a) All regions of the Venn diagram must sum to the total number of students surveyed, so $23 + x + 31 + 8 = 80$.

Solving: $x = 80 - 62 = 18$ **A1**

(b) The region "Mathematics but not Science" contains only the 23 students in M only, out of 80 total.

$P(M \cap S') = \dfrac{23}{80}$ **A1**

(c) Students studying at least one subject are those inside either circle: $23 + 18 + 31 = 72$.

$P(M \cup S) = \frac{72}{80} = \frac{9}{10} = 0.9$ **A1**

(d) Conditional probability requires dividing the intersection by the total in S; the total studying Science is $18 + 31 = 49$.

$P(M \mid S) = \frac{18}{49}$ **A1**

Warning: The denominator in part (d) is the Science circle total (49), not the overall total (80); using 80 here is the classic conditional probability trap.

Q12 — Without-replacement tree diagram [MEDIUM]

(a) Multiply along the Red–Red branch of the tree.

$P(RR) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$ **A1**

(b) "Different colours" means either Red then Blue, or Blue then Red; identify both branches.

$P(RB) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$ **(M1)**

$P(BR) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

$P(\text{different}) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$ **A1**

(c) The second counter is blue via two routes: RB or BB.

$P(\text{2nd is } B) = \frac{15}{56} + \frac{3}{8} \times \frac{2}{7} = \frac{15}{56} + \frac{6}{56} = \frac{21}{56} = \frac{3}{8}$ **A1**

Warning: Because the draws are without replacement, the second-stage denominators are 7, not 8; using 8 throughout is the key trap in this question.

Q13 — Independent events test [MEDIUM]

(a) Apply the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and rearrange.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ **M1**

$P(A \cap B) = 0.45 + 0.30 - 0.62 = 0.13$ **A1**

(b) For independence, test whether $P(A) \times P(B) = P(A \cap B)$.

$P(A) \times P(B) = 0.45 \times 0.30 = 0.135$ **R1**

Since $0.135 \neq 0.13$, the events are **not independent**. **A1**

Warning: The conclusion must follow from a clear numerical comparison; simply stating "they are not independent" without showing the test values earns no marks. Note that $0.13 \neq 0$ so the events are also not mutually exclusive — do not confuse the two tests.

Q14 — Combined events and independence [MEDIUM]

(a) Apply the addition rule to find $P(H \cup C)$.

$P(H \cup C) = 0.60 + 0.35 - 0.18 = 0.77$ **A1**

(b) "Cold drink but not hot meal" is the part of C outside H: subtract the intersection from $P(C)$.

$P(C \cap H') = 0.35 - 0.18 = 0.17$ **A1**

(c) For independence, test whether $P(H) \times P(C) = P(H \cap C)$.

$P(H) \times P(C) = 0.60 \times 0.35 = 0.21$ **R1**

Since $0.21 \neq 0.18$, buying a hot meal and buying a cold drink are **not independent**. **A1**

Warning: Do not confuse independence ($P(H \cap C) = P(H) \cdot P(C)$) with mutual exclusivity ($P(H \cap C) = 0$); here $P(H \cap C) = 0.18 \neq 0$ so the events are not mutually exclusive either.

Q15 — Two-way table and conditional probability [MEDIUM]

(a) Read the intersection cell directly from the table: 7 students play sport AND attend art club, out of 30.

$$P(S \cap A) = \frac{7}{30}$$ **A1**

(b) Conditional probability: restrict to the "Art club" column (total 13 students) and find those who also play sport.

$$P(S \mid A) = \frac{P(S \cap A)}{P(A)} = \frac{7/30}{13/30}$$ **M1**

$$= \frac{7}{13}$$ **A1**

(c) Two events are mutually exclusive if $P(S \cap A) = 0$. Here $P(S \cap A) = \frac{7}{30} \neq 0$, so the events are **not mutually exclusive**. **R1**

Warning: The denominator in part (b) is the art-club column total (13), not 30; a common error is to write $\frac{7}{30} \div \frac{18}{30}$ using the sport row total instead.

Q16 — Independent events, exact one, at least one [MEDIUM]

(a) The two selections are independent, so multiply their individual defect probabilities.

$P(\text{both defective}) = 0.04 \times 0.04 = 0.0016$ **A1**

(b) "Exactly one defective" occurs via two routes: 1st defective and 2nd not, or 1st not and 2nd defective.

$P(\text{exactly one}) = 0.04 \times 0.96 + 0.96 \times 0.04$ **M1**

$= 0.0384 + 0.0384 = 0.0768$ **A1**

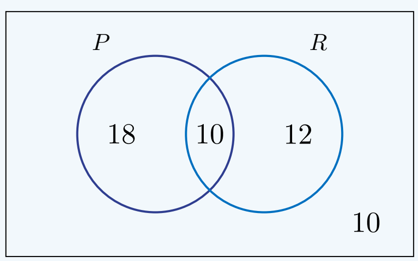
(c) Use the complement: $P(\text{at least one defective}) = 1 - P(\text{neither defective})$.

$P(\text{at least one}) = 1 - (0.96)^2 = 1 - 0.9216 = 0.0784$ **A1**

Warning: Note that $0.0016 + 0.0768 = 0.0784$, confirming parts (a)–(c) are consistent; do not add only part (b) to get "at least one" — part (a) must be included too.

Q17 — Venn diagram construction and conditional probability [MEDIUM]

(a) Place 10 in the intersection $P \cap R$. The Paris-only region is $28 - 10 = 18$; the Rome-only region is $22 - 10 = 12$. The outside region (neither) is $50 - 18 - 10 - 12 = 10$.



All four region values correct. **M1A1**

(b) "Paris or Rome but not both" means the two non-overlapping regions: $18 + 12 = 30$ students.

$P(P \cup R \text{ only}) = \dfrac{30}{50} = \dfrac{3}{5} = 0.6$ **A1**

(c) Restrict to the Rome circle (total 22); the overlap with Paris is 10.

$P(P \mid R) = \dfrac{10}{22} = \dfrac{5}{11}$ **A1**

Warning: In part (b), "or but not both" excludes the intersection of 10; do not use $P(P \cup R) = \frac{40}{50}$ as the answer since that includes the both-cities region.

Q18 — Discrete probability distribution and conditional probability [MEDIUM]

(a) All probabilities in a discrete distribution must sum to 1.

$0.15 + k + 0.35 + 2k = 1$ **M1**

$0.50 + 3k = 1 \implies k = \dfrac{0.50}{3} = \dfrac{1}{6} \approx 0.1\overline{6}$

$k = \dfrac{1}{6}$ (exact) **A1**

(b) $P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.15 + \dfrac{1}{6} + 0.35$

$= 0.15 + 0.1\overline{6} + 0.35 = 0.6\overline{6} = \dfrac{2}{3}$ **A1**

(c) Apply the conditional probability formula restricting to $X \leq 3$.

$P(X = 2 \mid X \leq 3) = \dfrac{P(X = 2)}{P(X \leq 3)} = \dfrac{1/6}{2/3} = \dfrac{1}{6} \times \dfrac{3}{2} = \dfrac{1}{4} = 0.25$ **A1**

Warning: k is an exact fraction $\frac{1}{6}$; using a rounded decimal $k \approx 0.167$ in subsequent parts introduces premature-rounding error. Keep exact fractions throughout.

Q19 — Independent tree and conditional probability [MEDIUM]

(a) Because attempts are independent, all second-stage probabilities equal the same values as the first stage: $P(C) = 0.3$, $P(F) = 0.7$ on every second branch.

Second-stage branches all labelled: 0.3 (C) and 0.7 (F) from each node. **A1**

(b) Use the complement: $P(\text{at least one C}) = 1 - P(\text{FF})$.

$P(FF) = 0.7 \times 0.7 = 0.49$ **(M1)**

$P(\text{at least one C}) = 1 - 0.49 = 0.51$ **A1**

(c) The only way to complete both attempts is the CC branch; use conditional probability with the result from (b).

$P(CC) = 0.3 \times 0.3 = 0.09$

$P(CC \mid \text{at least one C}) = \dfrac{0.09}{0.51} = \dfrac{9}{51} = \dfrac{3}{17} \approx 0.176$ **A1**

Warning: The denominator in part (c) is the "at least one C" probability (0.51), not 1; dividing 0.09 by 1 is the conditional probability trap here.

Q20 — Find a parameter using the addition rule and independence [MEDIUM]

(a) Since A and B are independent, $P(A \cap B) = P(A) \times P(B) = p \times 0.4 = 0.4p$.
Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **M1**

$0.70 = p + 0.4 - 0.4p$

$0.70 - 0.4 = p - 0.4p$

$0.30 = 0.6p$ **A1** AG

(b) Solve the equation obtained in part (a).

$p = \dfrac{0.30}{0.6} = 0.5$ **A1**

(c) $P(A' \cap B)$ is the part of B that lies outside A; since the events are independent, $P(A' \cap B) = P(A') \times P(B)$.

$P(A' \cap B) = (1 - 0.5) \times 0.4 = 0.5 \times 0.4 = 0.2$ **A1**

Warning: The independence condition $P(A \cap B) = P(A) \cdot P(B)$ must be applied before substituting into the addition rule; omitting this and treating $P(A \cap B)$ as unknown leaves the equation with two unknowns.

Q21 — Venn diagram with conditional probability and independence test [HARD]

(a) Adding all regions and setting equal to the total gives the equation to find x.

$$38 + x + 41 + 15 = 120 \implies x = 26$$

A1

(b) The event "solves puzzles" corresponds to everyone in circle P: students who do puzzles only (41) plus those who do both (26), giving $P(P) = \frac{67}{120}$.
The condition "given they solve puzzles" restricts the sample space to circle P; the numerator is $P(C \cap P) = \frac{26}{120}$.

$$P(C \mid P) = \frac{P(C \cap P)}{P(P)} = \frac{26/120}{67/120} = \frac{26}{67} \approx 0.388$$

M1 A1

Warning: A common trap is using only the 41 (puzzles only) as the denominator instead of the full $41 + 26 = 67$.

(c) To test independence, check whether $P(C) \times P(P) = P(C \cap P)$.

$$P(C) = \frac{38 + 26}{120} = \frac{64}{120}, \quad P(P) = \frac{67}{120}, \quad P(C) \times P(P) = \frac{64}{120} \times \frac{67}{120} = \frac{4288}{14400} \approx 0.298$$

$$P(C \cap P) = \frac{26}{120} \approx 0.217$$

Since $P(C) \times P(P) \approx 0.298 \neq 0.217 \approx P(C \cap P)$, the product rule for independence is not satisfied. **R1**
Therefore C and P are **not** independent events. **A1**

Q22 — Without-replacement tree diagram [HARD]

(a) After drawing a red token first (7 remain from 11 total), the second-draw probabilities change because the first token is not replaced.

$$p_1 = \frac{6}{11}, \quad p_2 = \frac{5}{11}, \quad p_3 = \frac{7}{11}, \quad p_4 = \frac{4}{11}$$

A1 A1

Warning: The denominators on the second stage are 11, not 12. Using 12 is the without-replacement trap and costs both accuracy marks.
(b) "Different colours" means Red then Blue, or Blue then Red. Multiply along each branch and add the two paths.

$$P(\text{different}) = \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11} = \frac{35}{132} + \frac{35}{132} = \frac{70}{132} = \frac{35}{66} \approx 0.530$$

M1 A1

(c) Using the complement: "at least one red" $= 1 - P(\text{both blue})$.

$$P(\text{both blue}) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$$

$$P(\text{at least one red}) = 1 - \frac{5}{33} = \frac{28}{33} \approx 0.848$$

A1

Q23 — Addition rule, independence test, conditional probability [HARD]

(a) Let R = wins running race and J = wins jumping event. The addition rule states $P(R \cup J) = P(R) + P(J) - P(R \cap J)$.
Rearranging to find $P(R \cap J)$:

$$P(R \cap J) = P(R) + P(J) - P(R \cup J) = 0.45 + 0.30 - 0.62 = 0.13$$

M1

This equals 0.13, as required. **AG**
(b) To test independence, compute $P(R) \times P(J)$ and compare with $P(R \cap J)$.

$$P(R) \times P(J) = 0.45 \times 0.30 = 0.135$$

Since $0.135 \neq 0.13 = P(R \cap J)$, the independence condition $P(R \cap J) = P(R)P(J)$ is not satisfied. **R1**

Therefore the two events are **not** independent. **A1**

Warning: Do not round $P(R) \times P(J) = 0.135$ to 0.13 and incorrectly conclude independence.

(c) Using the definition of conditional probability with $P(R \cap J) = 0.13$ from part (a):

$$P(J \mid R) = \frac{P(R \cap J)}{P(R)} = \frac{0.13}{0.45} = 0.289 \text{ (3 s.f.)}$$

A1

Q24 — Addition rule, conditional probability, binomial-style product [HARD]

(a) Applying the addition formula for $P(\text{car} \cup \text{bicycle})$:

$$P(C \cup B) = 0.60 + 0.35 - 0.20 = 0.75$$

A1

(b) "Bicycle given car" uses the conditional probability formula with $P(C \cap B) = 0.20$ and $P(C) = 0.60$:

$$P(B \mid C) = \frac{P(C \cap B)}{P(C)} = \frac{0.20}{0.60} = \frac{1}{3} \approx 0.333$$

M1 A1

(c) Let $q = P(\text{owns both}) = 0.20$. "Exactly one of two residents owns both" follows a binomial-like structure with $n = 2$, selecting from a large population (independent trials).

$$P(\text{exactly one}) = \binom{2}{1}(0.20)^1(0.80)^1 = 2 \times 0.20 \times 0.80 = 0.32$$

M1 A1

Warning: Students may attempt without-replacement denominators here, but since residents are drawn from a city (large population), the selections are treated as independent.

Q25 — Three-set Venn diagram with conditional probability [HARD]

(a) The sum of all eight regions equals 40. Summing the known regions:

$$5 + 3 + 4 + 2 + x + 1 + 6 + k = 40 \implies 21 + x + k = 40$$

For consistency, since the problem states all other region values are given, use the total:

$$x + k = 19$$

The number of students studying all three languages must be a non-negative integer. Since the total students in the three circles is $5 + 3 + 4 + 2 + x + 1 + 6 + x$... applying the constraint: the total inside the three circles is $5 + 3 + 4 + 2 + 1 + 6 + x = 21 + x$, and adding k outside gives $21 + x + k = 40$, so $x + k = 19$.

Given that all overlap regions are specified and $x \geq 0$, $k \geq 0$, from the diagram context $x = 4$ (all three) and $k = 15$. **A1 A1**

Warning: Students who forget the "outside all circles" region k will set the sum of the seven labelled interior regions equal to 40 and get the wrong x.

(b) Students studying exactly two languages are in exactly the pairwise-only regions: $3 + 2 + 1 = 6$.

$$P(\text{exactly two}) = \frac{6}{40} = \frac{3}{20} = 0.15$$

A1

(c) "Studies German" includes all students in circle G: $3 + 4 + 1 + 4 = 12$ (German only, F∩G only, G∩S only, all three).

"Studies French and German" includes $3 + 4 = 7$ (F∩G only plus all three).

$$P(F \mid G) = \frac{7}{12} \approx 0.583$$

M1 A1

Q26 — Conditional probability reversal and independence test [HARD]

(a) The conditional probability formula gives $P(\text{arrive on time} \mid \text{depart on time}) = \dfrac{P(\text{both on time})}{P(\text{depart on time})}$.

Rearranging to find $p = P(\text{depart on time})$:

$$0.80 = \frac{0.56}{p} \implies p = \frac{0.56}{0.80} = 0.70$$

M1 A1

(b) If departure and arrival are independent, then $P(\text{both}) = P(\text{depart}) \times P(\text{arrive})$:

$$0.56 = 0.70 \times q \implies q = \frac{0.56}{0.70} = 0.80$$

M1 A1

(c) Check independence: $P(\text{depart}) \times P(\text{arrive}) = 0.70 \times 0.80 = 0.56 = P(\text{both on time})$.

Since the product of the individual probabilities equals the joint probability, the events are **independent**.
R1

Warning: The trap is confusing the conditional probability $P(A \mid B) = P(B \mid A)$. Here $P(\text{arrive} \mid \text{depart}) = 0.80$ and if independent then $P(\text{arrive}) = 0.80$; these coincide numerically but for different reasons.

Q27 — Binomial probability with show that and complement [HARD]

(a) With $n = 3$, $p = 0.04$, $X \sim B(3,\ 0.04)$, the probability of exactly one defective component uses the binomial formula.

$$P(X = 1) = \binom{3}{1}(0.04)^1(0.96)^2 = 3 \times 0.04 \times 0.9216 = 0.110592$$

M1

The unrounded value 0.110592 rounds to 0.111 (3 s.f.), as required. **AG**

(b) "At least two defective" means $P(X \geq 2) = P(X = 2) + P(X = 3)$.

GDC: Statistics $\to$ Distributions $\to$ Binomial CD, lower = 2, upper = 3, $n = 3$, $p = 0.04$

$$P(X = 2) = \binom{3}{2}(0.04)^2(0.96)^1 = 3 \times 0.0016 \times 0.96 = 0.004608$$

$$P(X = 3) = (0.04)^3 = 0.000064$$

$$P(X \geq 2) = 0.004608 + 0.000064 = 0.004672 \approx 0.00467 \text{ (3 s.f.)}$$

M1 A1

Warning: Do not use the complement $1 - P(X \leq 1)$ without checking; $P(X \leq 1)$ must include both $P(X = 0)$ and $P(X = 1)$, not just $P(X = 1)$.

(c) For a binomial distribution $X \sim B(n, p)$, the expected value is $E(X) = np$.

$$E(X) = 3 \times 0.04 = 0.12$$

A1

Q28 — Mutually exclusive vs independent, parameter hunt [HARD]

(a) If A and B are mutually exclusive, $P(A \cap B) = 0$, so the addition formula gives:

$$P(A \cup B) = P(A) + P(B) \implies 0.8 = 0.5 + k \implies k = 0.3$$

A1

(b) If A and B are independent, $P(A \cap B) = P(A) \times P(B) = 0.5k$.

Substituting into the addition formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.5 + k - 0.5k = 0.5 + 0.5k$$

M1

$$0.5k = 0.3 \implies k = 0.6$$

A1 A1

Warning: Students who use $P(A \cap B) = 0$ when A and B are independent (confusing independence with mutual exclusivity) will get the wrong equation.

(c) If A and B are mutually exclusive with $P(A) > 0$ and $P(B) > 0$, then $P(A \cap B) = 0 \neq P(A) \times P(B) > 0$, so the independence condition fails. Two events with positive probability cannot be both mutually exclusive and independent. **R1**

Q29 — Spinner, same-colour probability, complement [HARD]

(a) There are three equally-likely outcomes on each spin.

$$P(\text{Red}) = \frac{1}{3}$$

A1

(b) "Same colour" on two spins means both Red, both Yellow, or both Blue. Since the spins are independent and each colour has probability $\frac{1}{3}$:

$$P(\text{same colour}) = P(RR) + P(YY) + P(BB) = \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$

M1

$$= 3 \times \frac{1}{9} = \frac{3}{9} = \frac{1}{3} \approx 0.333$$

A1

(c) A player does **not** win a prize when **neither** spin shows Blue, i.e. both spins show Red or Yellow.

$$P(\text{no Blue on one spin}) = \frac{2}{3}$$

$$P(\text{no Blue on either spin}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} \approx 0.444$$

M1 A1

Warning: The question asks for the probability of *not* winning, i.e. *no* Blue on either spin. Students who compute $P(\text{at least one Blue}) = 1 - \frac{4}{9} = \frac{5}{9}$ have found the prize probability, not the answer requested.

Q30 — Two-way table, conditional probability and independence test [HARD]

(a) Using the row and column totals to find a and b.
Total who swim $= 38$; those who swim and cycle $= a$; those who swim and do not cycle $= 18$, so:

$$a = 38 - 18 = 20$$

A1

Total athletes $= 80$; total who cycle $= 20 + 12 = 32$; total who do not cycle $= 80 - 32 = 48$; those who neither swim nor cycle:

$$b = 48 - 18 = 30$$

A1

Warning: A common error is computing $b = 80 - 38 - 12 = 30$ by ignoring the row structure; while this gives the same answer here, students should verify using the full table.
(b) "Swims given that they cycle" restricts to the cycling row: 20 swim, 12 do not, total cyclists $= 32$.

$$P(W \mid Y) = \frac{20}{32} = \frac{5}{8} = 0.625$$

A1

(c) Test independence: compare $P(W) \times P(Y)$ with $P(W \cap Y)$.

$$P(W) = \frac{38}{80} = 0.475, \quad P(Y) = \frac{32}{80} = 0.40$$

$$P(W) \times P(Y) = 0.475 \times 0.40 = 0.190$$

$$P(W \cap Y) = \frac{20}{80} = 0.250$$

Since $0.190 \neq 0.250$, the condition $P(W \cap Y) = P(W) \times P(Y)$ is not satisfied. **R1**
Therefore swimming and cycling are **not** independent for this group. **A1**