IB Math AI SL — Topic 4

Binomial Distribution

The Binomial Distribution · Calculating Binomial Probabilities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________________ Date:________________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

Key Formulae

Binomial probability (single value): $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$, where $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

Cumulative probability: $P(X \leq k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$ (use `binomcdf` on GDC)

Upper-tail conversion (boundary discipline): $P(X \geq k) = 1 - P(X \leq k-1)$ and $P(X > k) = 1 - P(X \leq k)$

Interior interval: $P(a \leq X \leq b) = P(X \leq b) - P(X \leq a-1)$

Expected value and variance: $E(X) = np$ Var$(X) = np(1-p)$ $SD(X) = \sqrt{np(1-p)}$

Reverse from mean/variance: given $E(X)$ and Var(X), solve $np = \mu$ and $np(1-p) = \sigma^2$ simultaneously to find n and p

Four conditions for $X \sim B(n,p)$: fixed number of trials n; only two outcomes (success/failure); constant probability p of success; trials are independent

GDC Tips

- **Exact probability $P(X = k)$:** Menu $\rightarrow$ Statistics $\rightarrow$ Distributions $\rightarrow$ Binomial Pdf — enter n, p, x value $= k$. Returns a single probability.
- **Cumulative probability $P(X \leq k)$:** Menu $\rightarrow$ Statistics $\rightarrow$ Distributions $\rightarrow$ Binomial Cdf — enter n, p, Lower $= 0$, Upper $= k$. Always write the boundary conversion before pressing EXE, e.g. $P(X \geq 5) = 1 - P(X \leq 4)$ so enter Upper $= 4$ then subtract from 1.
- **Interior interval $P(a \leq X \leq b)$:** use Binomial Cdf with Lower $= 0$, Upper $= b$, then subtract Binomial Cdf with Upper $= a - 1$. Equivalently set Lower $= a$ directly if your GDC supports it.
- **Threshold search (smallest n such that $P(X \geq 1) > c$):** evaluate $1 - (1 - p)^n$ for trial values of n using Binomial Cdf; test n and $n + 1$ explicitly and state which first exceeds the target.
- **Expected count in a sample:** compute $E(X) = np$ directly — no distribution menu needed; verify with List $\rightarrow$ Statistics if counts are given as a frequency table.

Common Mistakes to Avoid

1. **Off-by-one boundary error.** Writing $P(X \geq k) = 1 - P(X \leq k)$ instead of $1 - P(X \leq k - 1)$, or $P(X > k) = 1 - P(X \leq k - 1)$ instead of $1 - P(X \leq k)$. Always write the boundary conversion in full before entering values into the GDC, and check whether the inequality is strict ($>$) or non-strict ($\geq$).
2. **Confusing binompdf and binomcdf.** Using binompdf when a cumulative (range) probability is required, or vice versa. binompdf returns $P(X = k)$ only; binomcdf returns $P(X \leq k)$. Read the question carefully: "at most", "at least", "fewer than" all require binomcdf.
3. **Premature rounding of p in a chained problem.** When p is calculated from earlier data (e.g. a proportion or a normal probability), using a rounded value of p in the binomial GDC call introduces compounding error. Store the unrounded value in a GDC variable and use it directly.
4. **Forgetting to verify all four conditions.** Simply stating "it is binomial" without checking: (i) fixed n, (ii) two outcomes, (iii) constant p, (iv) independence. In context, independence is often the most doubtful condition (e.g. sampling without replacement from a small population) and must be explicitly addressed.
5. **Threshold search: not bracketing the integer answer.** Finding the first n that satisfies the inequality but failing to show that $n - 1$ does not satisfy it. Both values must be evaluated and compared with the target probability before the integer can be justified.
6. **Misidentifying $E(X) = np$ as a guaranteed outcome.** Confusing the expected value (a long-run average) with a certain or most-likely result. In context, a conclusion such as "exactly np items will be defective" is incorrect; the correct phrasing is "on average" or "in the long run".

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. **EASY** **Calculator** **[3 marks]**

A factory produces light bulbs. The probability that any one bulb is defective is 0.08. A quality-control inspector selects 15 bulbs at random. Let X be the number of defective bulbs in the sample.

(a) Write down the distribution of X, including the values of all parameters. [1]

(b) Find $P(X = 2)$. [2]

2. **EASY** **Calculator** **[3 marks]**

A seed germination trial shows that each seed has a probability of 0.75 of germinating, independently of all other seeds. A gardener plants 10 seeds. Let G be the number of seeds that germinate.

(a) Write down $E(G)$. [1]

(b) Find $P(G \leq 7)$. [2]

3. EASY Calculator **[3 marks]**

A basketball player has a probability of 0.6 of scoring a free throw. She takes 8 free throws in a practice session. Let S be the number of successful free throws.

(a) Write down the variance of S. [1]

(b) Find $P(S \geq 5)$. [2]

4. EASY Calculator **[3 marks]**

A multiple-choice quiz has 12 questions. Each question has 4 options, exactly one of which is correct. A student answers every question by guessing at random. Let C be the number of correct answers.
The probability distribution of C is shown in the table below.

c	0	1	2	3	4	$\cdots$
$P(C=c)$	0.0317	0.1267	0.2323	0.2581	0.1936	$\cdots$

(a) Write down the value of p, the probability of a correct answer on any one question. [1]

(b) Find $P(C \geq 4)$. [2]

5. EASY Calculator **[3 marks]**

A survey finds that 30% of adults in a city own a bicycle. A researcher randomly selects 20 adults. Let B be the number of adults in the sample who own a bicycle.

(a) Write down $E(B)$. [1]

(b) Find $P(B = 5)$. [2]

6. **EASY** **Calculator** **[3 marks]**

A technician repairs computers. The probability that any repair job is completed within one hour is 0.45. On a given day the technician has 9 jobs. Let W be the number of jobs completed within one hour. Assume each job is independent.
Find $P(W \leq 4)$.

7. **EASY** **Calculator** **[3 marks]**

A box contains a large number of marbles. It is known that 20% of the marbles are red. A child randomly selects 6 marbles, one at a time, replacing each marble before the next selection. Let R be the number of red marbles selected.

(a) State the distribution of R, giving the values of both parameters. [1]

(b) Find $P(R = 0)$. [2]

8. **EASY** **Calculator** **[3 marks]**

A soft-drink machine fills bottles. The probability that any bottle is under-filled is 0.05. A supervisor inspects a random sample of 18 bottles. Let U be the number of under-filled bottles in the sample.
The probability distribution of U is partially shown below.

u	0	1	2	3
$P(U = u)$	0.3972	0.3763	0.1683	0.0473

(a) Write down $E(U)$. [1]

(b) Find $P(U \leq 2)$. [2]

9. **EASY** **Calculator** **[3 marks]**

A hotel finds that each room booking is cancelled before arrival with probability 0.10, independently of other bookings. On a particular night, 7 rooms are booked. Let K be the number of cancellations.

(a) Write down Var(K). [1]

(b) Find $P(K \geq 1)$. [2]

10. **EASY** **Calculator** **[3 marks]**

A food-packaging line randomly inspects items. The probability that any item fails inspection is 0.12. A supervisor checks 11 items independently. Let F be the number of items that fail inspection.

(a) Write down $E(F)$. [1]

(b) Find $P(2 \leq F \leq 4)$. [2]

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator **[4 marks]**

A factory produces ceramic tiles. The probability that any single tile has a surface defect is 0.08. Tiles are inspected in batches of 25. Let X be the number of defective tiles in a randomly chosen batch.

(a) Write down the distribution of X, stating the values of all parameters. [1]

(b) Find the probability that a batch contains more than 3 defective tiles. [2]

(c) Find the expected number of defective tiles in a batch. [1]

12. MEDIUM Calculator **[4 marks]**

At a university, 35% of students own a bicycle. A random sample of 12 students is selected. Let B be the number of students in the sample who own a bicycle.

(a) Find the probability that exactly 5 students own a bicycle. [2]

(b) Find the probability that at least 4 students own a bicycle. [2]

13. MEDIUM Calculator **[4 marks]**

A biased coin has probability p of landing on heads. The coin is tossed 20 times. The expected number of heads is 7.

(a) Find the value of p. [2]

(b) Find the variance of the number of heads. [2]

14. MEDIUM Calculator **[4 marks]**

A seed supplier claims that 72% of a particular variety of flower seed will germinate. A gardener plants 18 seeds. Let G be the number of seeds that germinate.

Number germinating, g	≤ 10	11	12	≥ 13
$P(G = g)$	0.1014	0.1157	0.1736	see (b)

(a) Find the probability that fewer than 13 seeds germinate. [2]

(b) Find the probability that at least 13 seeds germinate. [2]

15. **MEDIUM** **Calculator** **[4 marks]**

In a large city, the probability that a randomly selected household recycles regularly is 0.45. A researcher surveys 16 randomly selected households. Let R be the number of households that recycle regularly.

(a) Find $P(R \geq 8)$. [2]

(b) Find the probability that between 6 and 10 households, inclusive, recycle regularly. [2]

16. **MEDIUM** **Calculator** **[4 marks]**

A sports analyst models the number of penalty shots saved by a goalkeeper in a sequence of 10 penalty attempts. The probability the goalkeeper saves any single penalty is 0.3, independently of others. Let S be the number of penalties saved.

(a) Find the probability that the goalkeeper saves none of the 10 penalties. [2]

(b) Find the probability that the goalkeeper saves at most 2 penalties. [2]

17. MEDIUM Calculator **[4 marks]**

The probability that a passenger at a particular airport check-in desk needs extra assistance is 0.12. A check-in agent serves n passengers in one shift. It is found that the expected number of passengers needing extra assistance is 3.6.

(a) Find the value of n. [2]

(b) Using your value of n, find the probability that more than 5 passengers need extra assistance. [2]

18. MEDIUM Calculator **[4 marks]**

A tech company tests batches of 25 memory cards. The probability that any single card is faulty is 0.10. The number of faulty cards in a batch is modelled by $F \sim B(25, 0.10)$.

f	0	1	2	3	≥ 4
$P(F = f)$	0.0718	0.1994	0.2659	0.2265	?

(a) Show that $P(F \leq 3) = 0.7636$, correct to four significant figures. [2]

(b) Hence find the probability that a batch contains 4 or more faulty cards. [1]

(c) Find the expected number of faulty cards in a batch. [1]

19. MEDIUM Calculator **[4 marks]**

A survey finds that 28% of teenagers in a country play a musical instrument. A random sample of 15 teenagers is chosen. Let M be the number who play a musical instrument.

(a) Find the probability that exactly 3 teenagers play a musical instrument. [2]

(b) Given that at least one teenager plays a musical instrument, find the probability that exactly 3 play a musical instrument. [2]

20. MEDIUM Calculator **[4 marks]**

A restaurant manager records whether each customer orders dessert. The probability that any customer orders dessert is 0.38, independently of others. One evening 20 customers dine at the restaurant. Let D be the number who order dessert.

(a) Find the probability that fewer than half of the 20 customers order dessert. [2]

(b) Find the most likely number of customers to order dessert, justifying your answer. [2]

Section C — Hard HARD (10 questions $\times$ 5 marks = 50 marks)

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21. HARD Calculator **[5 marks]**

A quality-control engineer inspects circuit boards produced at a factory. The probability that any single board is defective is 0.08. The engineer selects a random sample of 25 boards. Let X be the number of defective boards in the sample.

(a) Show that $E(X) = 2$ and find Var(X). [2]

(b) Find $P(X \geq 4)$. [1]

(c) The engineer takes a second independent sample of 25 boards. Find the probability that both samples contain fewer than 4 defective boards. [2]

22. HARD Calculator **[5 marks]**

A seed supplier claims that 72% of seeds of a particular variety will germinate. A gardener plants n seeds, chosen at random. Let Y be the number of seeds that germinate. The gardener wants the probability that at least one seed germinates to exceed 0.9999.

(a) Write down a suitable model for Y, stating one assumption required. [1]

(b) Find the smallest value of n such that $P(Y \geq 1) > 0.9999$. [4]

23. HARD Calculator **[5 marks]**

At a hospital, records show that 35% of patients admitted for a routine procedure require a follow-up appointment. On one particular day, 18 such patients are admitted. Let W be the number of patients who require a follow-up appointment.

(a) Find $P(W = 5)$. [1]

(b) Find $P(4 \leq W \leq 9)$. [2]

(c) Given that $P(W \leq c) > 0.8$, find the smallest integer value of c. [2]

24. HARD Calculator **[5 marks]**

A factory produces light bulbs. Past data show that the mean number of faulty bulbs per box of 40 is 3.2.

(a) Show that the probability that a randomly selected bulb is faulty is $p = 0.08$. [1]

(b) Let F be the number of faulty bulbs in a box of 40. Using $p = 0.08$, find $P(F \leq 2)$. [1]

(c) A shop receives 12 boxes. Find the probability that exactly 3 of these boxes contain at most 2 faulty bulbs. [3]

25. HARD Calculator **[5 marks]**

The probability that a randomly chosen customer at a coffee shop orders a large drink is p. In a random sample of 10 customers, the variance of the number of customers ordering a large drink is 1.96.

(a) Show that p satisfies $10p(1-p) = 1.96$, and hence find the two possible values of p. [3]

(b) Given that more than half the customers are expected to order a large drink, state which value of p applies. [1]

(c) Using your value of p, find $P(X \geq 7)$, where X is the number of customers ordering a large drink in a sample of 10. [1]

26. HARD Calculator **[5 marks]**

A biologist is studying a species of frog. The probability that a randomly chosen egg from this species hatches successfully is 0.62. A researcher collects a clutch of 30 eggs. Let H be the number of eggs that hatch successfully.

(a) Find $P(H > 20)$. [2]

(b) The researcher collects 8 clutches, each of 30 eggs. Find the expected number of clutches in which more than 20 eggs hatch. [2]

(c) State one reason why the binomial model may not be appropriate in this context. [1]

27. HARD Calculator **[5 marks]**

In a large city, 42% of adults own a bicycle. A researcher surveys a random sample of adults.

(a) In a sample of 20 adults, let B be the number who own a bicycle.

(i) Write down the distribution of B. [1]

(ii) Find $P(B \leq 7)$. [1]

(b) The researcher increases the sample size to n adults. Find the smallest value of n such that $P(B \geq 1) > 0.999$. [3]

28. HARD Calculator **[5 marks]**

A game at a charity fair costs $2 to play. A player rolls a fair six-sided die; if the result is a 5 or a 6, the player wins $5 (and gets their $2 entry fee back), otherwise they lose their entry fee. Let W be the number of times a player wins in n plays of the game.

Outcome	Probability	Net gain ($)
Win	$\frac{1}{3}$	$+5$
Lose	$\frac{2}{3}$	-2

(a) Show that the expected net gain per single play is $\$\frac{1}{3}$. [1]

(b) A player decides to keep playing until the expected total net gain exceeds $10. Find the minimum number of plays required. [2]

(c) Using your value of n, find the probability that the player wins more than half of their plays. [2]

29. HARD Calculator **[5 marks]**

The number of passengers, T, who miss a connecting flight at an airport on any given day follows a binomial distribution. Each of the 15 passengers on a feeder flight independently has a probability of 0.18 of missing their connection.

(a) Find $P(T = 0)$. [1]

(b) Find $P(T \geq 3)$. [2]

(c) The airport manager states: "On at least 90% of days, fewer than k passengers miss their connection." Find the smallest integer value of k for which this statement is correct. [2]

30. **HARD** **Calculator** **[5 marks]**

A statistician models the number of goals scored by a football team per match. In each of n independent penalty shootout attempts, the probability that the team scores is 0.76. Let G be the number of goals scored in $n = 20$ attempts.

(a) Find $P(G = 15)$. [1]

(b) Find $P(13 \leq G \leq 17)$. [1]

(c) The team is said to have a "strong performance" if they score at least m goals, where $P(G \geq m) < 0.25$. Find the smallest value of m satisfying this condition, and justify your answer. [3]

Mark Schemes

Q1 — Defective Light Bulbs EASY

(a) 15 independent trials, constant $p = 0.08$: $X \sim B(15, 0.08)$ **A1**

(b) Using `binompdf(15, 0.08, 2)` **M1**

$P(X = 2) = 0.22731\ldots \approx 0.227$ **A1**

Q2 — Seed Germination EASY

(a) $E(G) = np = 10 \times 0.75 = 7.5$ **A1**

(b) Using `binomcdf(10, 0.75, 7)` **M1**

$P(G \leq 7) = 0.47441\ldots \approx 0.474$ **A1**

Q3 — Free Throws EASY

(a) Var$(S) = np(1-p) = 8 \times 0.6 \times 0.4 = 1.92$ **A1**

(b) $P(S \geq 5) = 1 - P(S \leq 4)$ **M1**

Using `binomcdf(8, 0.6, 4)`$= 0.40591\ldots$
$P(S \geq 5) = 1 - 0.40591\ldots = 0.59409\ldots \approx 0.594$ **A1**

Q4 — Multiple-Choice Guessing EASY

(a) 4 equally likely options: $p = 0.25$ **A1**

(b) $P(C \geq 4) = 1 - P(C \leq 3)$ **M1**

Using `binomcdf(12, 0.25, 3)`$= 0.64878\ldots$
$P(C \geq 4) = 1 - 0.64878\ldots = 0.35122\ldots \approx 0.351$ **A1**

Q5 — Bicycle Ownership EASY

(a) $E(B) = np = 20 \times 0.30 = 6$ **A1**

(b) Using `binompdf(20, 0.30, 5)` **M1**

$P(B = 5) = 0.17886\ldots \approx 0.179$ **A1**

Q6 — Computer Repair Jobs **EASY**

$W \sim B(9, 0.45)$; boundary $W \leq 4$ already in cdf form	**M1**
Using `binomcdf(9, 0.45, 4)`	**M1**
$P(W \leq 4) = 0.62142\ldots \approx 0.621$	**A1**

Q7 — Red Marbles with Replacement **EASY**

(a) Constant $p = 0.20$, replacement gives independence: $R \sim B(6, 0.20)$	**A1**
(b) Using `binompdf(6, 0.20, 0)`	**M1**
$P(R = 0) = 0.26214\ldots \approx 0.262$	**A1**

Q8 — Under-Filled Bottles **EASY**

(a) $E(U) = np = 18 \times 0.05 = 0.9$	**A1**
(b) $P(U \leq 2)$ already in cdf form	**M1**
Using `binomcdf(18, 0.05, 2)` $= 0.94187\ldots$, consistent with the table sum $0.3972+0.3763+0.1683 = 0.9418$	
$P(U \leq 2) = 0.942$ (3 s.f.)	**A1**

Q9 — Hotel Cancellations **EASY**

(a) Var$(K) = np(1-p) = 7 \times 0.10 \times 0.90 = 0.63$	**A1**
(b) $P(K \geq 1) = 1 - P(K = 0)$	**M1**
Using `binompdf(7, 0.10, 0)`$= 0.47830\ldots$	
$P(K \geq 1) = 1 - 0.47830\ldots = 0.52170\ldots \approx 0.522$	**A1**

Q10 — Food-Packaging Inspection **EASY**

(a) $E(F) = np = 11 \times 0.12 = 1.32$	**A1**
(b) $P(2 \leq F \leq 4) = P(F \leq 4) - P(F \leq 1)$	**M1**
Using `binomcdf(11, 0.12, 4)`$= 0.99386\ldots$ and `binomcdf(11, 0.12, 1)`$= 0.61270\ldots$	
$P(2 \leq F \leq 4) = 0.99386\ldots - 0.61270\ldots = 0.38116\ldots \approx 0.381$	**A1**

Q11 — Ceramic Tile Defects MEDIUM

(a) Fixed $n = 25$, constant $p = 0.08$, independent inspections: $X \sim B(25, 0.08)$ **A1**

(b) $P(X > 3) = 1 - P(X \leq 3)$ **M1**

Using `binomcdf(25, 0.08, 3)`$= 0.86491\ldots$
$P(X > 3) = 1 - 0.86491\ldots = 0.13509\ldots \approx 0.135$ **A1**

(c) $E(X) = np = 25 \times 0.08 = 2$ **A1**

Q12 — Bicycle Ownership at University MEDIUM

(a) Using `binompdf(12, 0.35, 5)` **M1**

$P(B = 5) = 0.20392\ldots \approx 0.204$ **A1**

(b) $P(B \geq 4) = 1 - P(B \leq 3)$ **M1**

Using `binomcdf(12, 0.35, 3)`$= 0.34665\ldots$
$P(B \geq 4) = 1 - 0.34665\ldots = 0.65335\ldots \approx 0.653$ **A1**

Q13 — Reverse Binomial: Find p Then Variance MEDIUM

(a) $E(X) = np = 7 \implies 20p = 7 \implies p = 0.35$ **M1 A1**

(b) Var$(X) = np(1-p) = 20 \times 0.35 \times 0.65 = 4.55$ **M1 A1**

Q14 — Complementary Probabilities from a Table MEDIUM

(a) "Fewer than 13" means $G \leq 12 = P(G \leq 10) + P(G = 11) + P(G = 12)$ **M1**

Using the table: $P(G \leq 12) = 0.1014 + 0.1157 + 0.1736 = 0.3907$
Confirmed with `binomcdf(18, 0.72, 12)`$= 0.39074\ldots \approx 0.391$ **A1**

(b) $P(G \geq 13) = 1 - P(G \leq 12) = 1 - 0.39074\ldots = 0.60926\ldots \approx 0.609$ **M1 A1**

Q15 — Binomial Cdf: Inclusive Interval MEDIUM

(a) $P(R \geq 8) = 1 - P(R \leq 7)$ **M1**

Using `binomcdf(16, 0.45, 7)`$= 0.56290\ldots$
$P(R \geq 8) = 1 - 0.56290\ldots = 0.43710\ldots \approx 0.437$ **A1**

(b) $P(6 \leq R \leq 10) = P(R \leq 10) - P(R \leq 5)$ **M1**

Using `binomcdf(16, 0.45, 10)`$= 0.95138\ldots$ and `binomcdf(16, 0.45, 5)`$= 0.19760\ldots$
$P(6 \leq R \leq 10) = 0.95138\ldots - 0.19760\ldots = 0.75378\ldots \approx 0.754$ **A1**

Q16 — At-Least-One Complement; At-Most Boundary MEDIUM

(a) $S \sim B(10, 0.3)$; $P(S=0) = (0.7)^{10}$ **M1**

$P(S=0) = 0.02825\ldots \approx 0.0282$ **A1**

(b) $P(S \leq 2)$ already in cdf form **M1**

Using `binomcdf(10, 0.3, 2)`: $P(S \leq 2) = 0.38278\ldots \approx 0.383$ **A1**

Q17 — Reverse Binomial Find n; Then Tail Probability MEDIUM

(a) $np = 3.6 \implies n \times 0.12 = 3.6 \implies n = 30$ **M1 A1**

(b) With $n = 30$: $P(X > 5) = 1 - P(X \leq 5)$ **M1**

Using `binomcdf(30, 0.12, 5)`$= 0.85692\ldots$
$P(X > 5) = 1 - 0.85692\ldots = 0.14308\ldots \approx 0.143$ **A1**

Q18 — Show That AG; Complement; Expected Value MEDIUM

(a) Adding the given individual probabilities for $f = 0, 1, 2, 3$: **M1**

$P(F \leq 3) = 0.0718 + 0.1994 + 0.2659 + 0.2265 = 0.7636$
Confirmed with `binomcdf(25, 0.10, 3)`$= 0.76359\ldots \approx 0.7636$ (4 s.f.) **A1 AG**

(b) $P(F \geq 4) = 1 - 0.7636 = 0.23641\ldots \approx 0.236$ **A1**

(c) $E(F) = np = 25 \times 0.10 = 2.5$ **A1**

Q19 — Conditional Probability with Binomial MEDIUM

(a) Using `binompdf(15, 0.28, 3)` **M1**

$P(M=3) = 0.19385\ldots \approx 0.194$ **A1**

(b) $P(M=3 \mid M \geq 1) = \dfrac{P(M=3)}{P(M \geq 1)}$; first find $P(M \geq 1) = 1 - P(M=0)$ **M1**

Using `binompdf(15, 0.28, 0)`$= (0.72)^{15} = 0.00724\ldots$
$P(M \geq 1) = 1 - 0.00724\ldots = 0.99276\ldots$
$P(M=3 \mid M \geq 1) = \dfrac{0.19385\ldots}{0.99276\ldots} = 0.19527\ldots \approx 0.195$ **A1**

Q20 — Tail Probability; Mode by Inspection MEDIUM

(a) $D \sim B(20, 0.38)$; "fewer than half of 20" means $D \leq 9$ **M1**

Using `binomcdf(20, 0.38, 9)`: $P(D \leq 9) = 0.81032\ldots \approx 0.810$ **A1**

(b) Compare $d = 7$ and $d = 8$ near $np = 7.6$ **M1**

`binompdf(20,0.38,7)`$= 0.17742\ldots$; `binompdf(20,0.38,8)`$= 0.17670\ldots$
Since $P(D = 7) > P(D = 8)$, the most likely number of customers to order dessert is **7**. **A1**

Q21 — Defective Circuit Boards Two-Sample Fusion HARD

(a) $E(X) = np = 25 \times 0.08 = 2$ **AG**

Var$(X) = np(1 - p) = 25 \times 0.08 \times 0.92 = 1.84$ **A1**

(b) $P(X \geq 4) = 1 - P(X \leq 3)$ **M1**

Using `binomcdf(25, 0.08, 3)`$= 0.86491\ldots$
$P(X \geq 4) = 1 - 0.86491\ldots = 0.13509\ldots \approx 0.135$ **A1**

(c) Let A be "a sample contains fewer than 4 defectives", $P(A) = P(X \leq 3) = 0.86491\ldots$ **M1**

Samples independent: required probability $= [P(X \leq 3)]^2 = (0.86491\ldots)^2 = 0.74807\ldots \approx 0.748$ **A1**

Q22 — Seed Germination Threshold Search HARD

(a) $Y \sim B(n, 0.72)$; assumption: each seed germinates independently of every other seed **A1**

(b) $P(Y \geq 1) > 0.9999 \implies P(Y = 0) < 0.0001$; $P(Y = 0) = (0.28)^n$ **M1**

Test bracketing values:
$n = 7$: $(0.28)^7 = 0.0001349\ldots > 0.0001$ (does *not* satisfy) **M1**
$n = 8$: $(0.28)^8 = 0.0000378\ldots < 0.0001$ (satisfies) **M1**

Since $(0.28)^7 > 0.0001$ but $(0.28)^8 < 0.0001$, the smallest value is $n = 8$. **A1**

Q23 — Hospital Follow-Up Appointments Threshold Search HARD

(a) $W \sim B(18, 0.35)$; using `binompdf(18, 0.35, 5)` **M1**

$P(W = 5) = 0.16638\ldots \approx 0.166$ **A1**

(b) $P(4 \leq W \leq 9) = P(W \leq 9) - P(W \leq 3)$ **M1**

Using `binomcdf(18, 0.35, 9)`$= 0.94031\ldots$ and `binomcdf(18, 0.35, 3)`$= 0.07827\ldots$
$P(4 \leq W \leq 9) = 0.94031\ldots - 0.07827\ldots = 0.86204\ldots \approx 0.862$ **A1**

(c) Evaluate successive c: **M1**

$c = 7$: $P(W \leq 7) = 0.72828\ldots < 0.8$; $\quad c = 8$: $P(W \leq 8) = 0.86094\ldots > 0.8$
The smallest integer value is $c = 8$. **A1**

Q24 — Light Bulb Quality Control Two-Technique Fusion HARD

(a) $E(F) = np = 3.2, n = 40 \implies p = \dfrac{3.2}{40} = 0.08$ **AG**

(b) $F \sim B(40, 0.08)$; using `binomcdf(40, 0.08, 2)` **M1**

$P(F \leq 2) = 0.36945\ldots \approx 0.369$ **A1**

(c) Let $q = P(F \leq 2) = 0.36945\ldots$ (unrounded); $Q \sim B(12, q)$; require $P(Q = 3)$ **M1**

Using `binompdf(12, 0.36945, 3)` **M1**

$P(Q = 3) = 0.17481\ldots \approx 0.175$ **A1**

Q25 — Coffee Shop Large Drinks Reverse Parameter Hunt HARD

(a) Var$(X) = np(1-p) = 10p(1-p) = 1.96$ **AG**

$10p - 10p^2 = 1.96 \implies p^2 - p + 0.196 = 0$ **M1**

$$p = \frac{1 \pm \sqrt{1 - 4(0.196)}}{2} = \frac{1 \pm 0.46476\ldots}{2}$$

$p = 0.73238\ldots \approx 0.732$ or $p = 0.26762\ldots \approx 0.268$ **A1**

(b) $E(X) = 10p > 5$ (more than half) requires $p > 0.5$, so $p = 0.732$ **A1**

(c) With $p = 0.73238\ldots$: $P(X \geq 7) = 1 - P(X \leq 6)$ **M1**

Using `binomcdf(10, 0.73238, 6)`$= 0.26664\ldots$
$P(X \geq 7) = 1 - 0.26664\ldots = 0.73336\ldots \approx 0.733$ **A1**

Q26 — Frog Egg Hatching Expected Count and Model Validity HARD

(a) $H \sim B(30, 0.62)$; $P(H > 20) = 1 - P(H \leq 20)$ **M1**

Using `binomcdf(30, 0.62, 20)`$= 0.75991\ldots$
$P(H > 20) = 1 - 0.75991\ldots = 0.24009\ldots \approx 0.240$ **A1**

(b) Expected clutches (out of 8) $= 8 \times P(H > 20)$ **M1**

$= 8 \times 0.24009\ldots = 1.9207\ldots \approx 1.92$ clutches **A1**

(c) Eggs within the same clutch share genetic material and environmental conditions, so hatching probability may not be constant, or eggs within a clutch may not be independent. **A1**

Q27 — Bicycle Ownership Distribution and Threshold Search HARD

(a)(i) $B \sim B(20, 0.42)$ **A1**

(a)(ii) Using `binomcdf(20, 0.42, 7)`: $P(B \leq 7) = 0.34613\ldots \approx 0.346$ **A1**

(b) $P(B \geq 1) > 0.999 \implies P(B = 0) = (0.58)^n < 0.001$ **M1**

Test bracketing values:
$n = 12$: $(0.58)^{12} = 0.0014492\ldots > 0.001$ (does *not* satisfy) **M1**
$n = 13$: $(0.58)^{13} = 0.0008406\ldots < 0.001$ (satisfies) **M1**
Since $(0.58)^{12} > 0.001$ but $(0.58)^{13} < 0.001$, the smallest value is $n = 13$. **A1**

Q28 — Charity Fair Game Expected Gain and Binomial Probability HARD

(a) $E = \frac{1}{3}(+5) + \frac{2}{3}(-2) = \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$ **AG**

(b) After n plays, expected total net gain $= \dfrac{n}{3}$; require $\dfrac{n}{3} > 10 \implies n > 30$ **M1**

Since n must be a whole number, the minimum is $n = 31$. **A1**

(c) With $n = 31, p = \frac{1}{3}$: $W \sim B(31, \frac{1}{3})$. "More than half" of 31 means $W \geq 16$ **M1**

$P(W \geq 16) = 1 - P(W \leq 15)$; using `binomcdf(31, 1/3, 15)`$= 0.97298\ldots$
$P(W \geq 16) = 1 - 0.97298\ldots = 0.02702\ldots \approx 0.0270$ **A1**

Q29 — Airport Missed Connections Threshold and Justified Decision HARD

(a) $T \sim B(15, 0.18)$; using `binompdf(15, 0.18, 0)`$= (0.82)^{15}$ **M1**

$P(T = 0) = 0.05096\ldots \approx 0.0510$ **A1**

(b) $P(T \geq 3) = 1 - P(T \leq 2)$ **M1**

Using `binomcdf(15, 0.18, 2)`$= 0.47656\ldots$
$P(T \geq 3) = 1 - 0.47656\ldots = 0.52344\ldots \approx 0.523$ **A1**

(c) Require smallest k with $P(T \leq k - 1) > 0.90$ **M1**

$P(T \leq 4) = 0.88331\ldots < 0.90$; $\quad P(T \leq 5) = 0.96130\ldots > 0.90$
So $k - 1 = 5$, giving $k = 6$. **A1**

Q30 — Penalty Shootout Goals Threshold with Justified Decision HARD

(a) $G \sim B(20, 0.76)$; using `binompdf(20, 0.76, 15)` **M1**

$P(G = 15) = 0.20124\ldots \approx 0.201$ **A1**

(b) $P(13 \leq G \leq 17) = P(G \leq 17) - P(G \leq 12)$ **M1**

Using `binomcdf(20, 0.76, 17)`$= 0.89145\ldots$ and `binomcdf(20, 0.76, 12)`$= 0.08349\ldots$
$P(13 \leq G \leq 17) = 0.89145\ldots - 0.08349\ldots = 0.80796\ldots \approx 0.808$ **A1**

(c) Require smallest m with $P(G \geq m) < 0.25$, i.e. $P(G \leq m - 1) > 0.75$ **M1**

$m = 17$: $P(G \geq 17) = 1 - P(G \leq 16) = 1 - 0.74307\ldots = 0.25693\ldots > 0.25$
$m = 18$: $P(G \geq 18) = 1 - P(G \leq 17) = 1 - 0.89145\ldots = 0.10855\ldots < 0.25$ **M1**

Since $P(G \geq 17) > 0.25$ but $P(G \geq 18) < 0.25$, the smallest value of m is **18**. **R1**

Common Mistakes to Avoid

1. **Off-by-one boundary error.** Writing $P(X \geq k) = 1 - P(X \leq k)$ instead of $1 - P(X \leq k-1)$. Always write the boundary conversion in full before entering values into the GDC.
2. **Confusing binompdf and binomcdf.** binompdf returns $P(X = k)$ only; binomcdf returns $P(X \leq k)$. "At most", "at least", "fewer than" all require binomcdf.
3. **Premature rounding in a chained problem.** When a probability from an earlier part feeds a later binomial call (as in Q24(c)), store the unrounded value and use it directly — rounding first introduces compounding error.
4. **Not bracketing the integer answer in a threshold search.** Always evaluate both the candidate value and the value immediately below it, and compare both against the target, before stating the smallest integer that works.
5. **Misidentifying $E(X) = np$ as a guaranteed outcome.** The expected value is a long-run average, not a certain result; a conclusion such as "exactly np items will be defective" is incorrect.
6. **Confusing the mode with the mean.** The most likely value must be found by comparing $P(X = \lfloor np \rfloor)$ with its neighbours directly — rounding np to the nearest integer without checking both neighbouring probabilities is not sufficient justification.