IB Math AI SL — Topic 4

Normal Distribution

The Normal Curve · Normal Calculations & Inverse Normal

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________ Date:____________

Score:________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Normal distribution notation:** $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance (note: σ is the standard deviation).
- **Standardisation (z-score):** $z = \dfrac{x-\mu}{\sigma}$, so that $Z \sim N(0,1)$.
- **Symmetry property:** $P(X < \mu) = P(X > \mu) = 0.5$ for any normal distribution.
- **Complement rule:** $P(X > a) = 1 - P(X < a)$.
- **Interval probability:** $P(a < X < b) = P(X < b) - P(X < a)$.
- **Symmetry about the mean:** $P(\mu - k < X < \mu + k) = 2\,P(\mu < X < \mu + k)$ for any $k > 0$.
- **Expected count from a sample of n:** Expected count $= n \times P(X \in \text{region})$.

GDC Tips

- **Computing $P(a < X < b)$:** `Menu` $\to$ `Statistics` $\to$ `Distributions` $\to$ `Normal CDF`; enter lower $= a$, upper $= b$, μ, σ. For $P(X < a)$ use lower $= -10^{99}$; for $P(X > a)$ use upper $= 10^{99}$.
- **Inverse normal — finding a threshold x from a given probability:** `Menu` $\to$ `Statistics` $\to$ `Distributions` $\to$ `Inverse Normal`; enter the cumulative area to the **LEFT**, μ, σ. Always confirm which tail the question specifies before entering the area.
- **Tail direction for inverse normal:** If the question asks for the top $q\%$ (e.g. "fastest 5%"), the cumulative LEFT area to enter is $1 - 0.01q$, *not* $0.01q$.
- **Finding an unknown μ or σ from one probability condition:** Use `Menu` $\to$ `Calculator` $\to$ `Numerical Solve` (or `nSolve`); set up `normalcdf`$(a, b, \mu, \sigma) = p$ and solve for the unknown parameter directly — algebraic standardisation is not required.
- **Checking symmetry on the GDC:** Verify $P(X < \mu) = 0.5$ by running `Normal CDF` with lower $= -10^{99}$, upper $= \mu$; the result should display 0.5 exactly, confirming correct entry of μ and σ.

Common Mistakes to Avoid

1. **Wrong tail in inverse normal.** Students enter the given percentage directly as the LEFT-tail area even when the question describes the upper tail (e.g. "top 8%" or "fastest times"). Always ask: is the shaded region on the LEFT or RIGHT? For an upper-tail probability p, enter $1 - p$ into `Inverse Normal`.
2. **Confusing σ^2 and σ when entering the GDC.** The notation $X \sim N(50, 16)$ means $\mu = 50$ and $\sigma^2 = 16$, so $\sigma = 4$. Entering $\sigma = 16$ into `normalcdf` or `Inverse Normal` gives a completely wrong answer with no error message.
3. **Premature rounding in chained parts.** Rounding an intermediate probability (e.g. to 2 decimal places) before multiplying by n to find an expected count can introduce a large error in the final answer. Always carry the full unrounded GDC value through to the next step.
4. **Forgetting the complement for "greater than" probabilities.** Using `Normal CDF` with lower $= a$ and upper $= 10^{99}$ is correct, but students sometimes instead compute $P(X < a)$ and forget to subtract from 1, or subtract from the wrong value.
5. **Assertive conclusions after a probability comparison.** A conclusion such as "therefore it is certain that ..." is wrong. Probability-based conclusions must be non-assertive: e.g. "there is evidence to suggest ..." or "it is likely that ...". The R1 mark requires the comparison of values first, then a hedged contextual statement.
6. **Treating the expected count as exact without appropriate rounding.** An expected count of people, items, or events must be rounded to a whole number (or left as a decimal only if the question says "estimate"). Writing 17.3 students as the final answer to "how many students do you expect" loses the accuracy mark.

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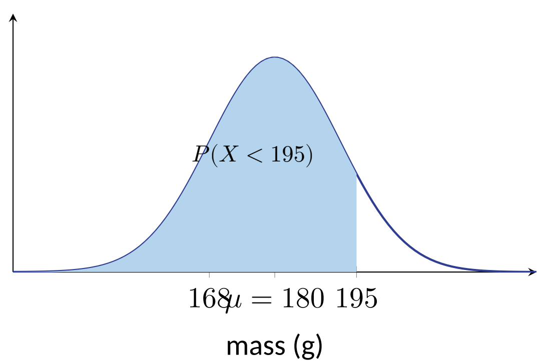
Section A — Normal Distribution

EASY

1. The mass of apples harvested at a farm in New Zealand is normally distributed with mean $\mu = 180$ g and standard deviation $\sigma = 12$ g. Let X be the mass, in grams, of a randomly selected apple.
Find $P(X < 195)$.

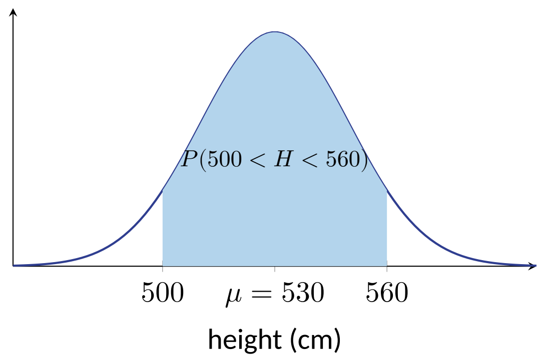


[3 marks]

2. The time, T minutes, that customers wait in a queue at a bank in Singapore is normally distributed with mean $\mu = 8$ minutes and standard deviation $\sigma = 2$ minutes.
Find $P(T > 11)$.

[3 marks]

3. The height, H cm, of adult male giraffes in a wildlife reserve in Kenya is normally distributed with mean $\mu = 530$ cm and standard deviation $\sigma = 20$ cm.
Find $P(500 < H < 560)$.



[3 marks]

4. The volume of coffee, V ml, dispensed by a machine in a café in Italy is normally distributed with mean $\mu = 250$ ml and standard deviation $\sigma = 5$ ml.

(a) Write down $P(V < 250)$. **[1 marks]**

(b) Find $P(V > 258)$. **[2 marks]**

5. The score, S, achieved by students in a mathematics test at a school in Canada is normally distributed with mean $\mu = 62$ and standard deviation $\sigma = 8$.
Find the value of s such that $P(S < s) = 0.90$.

[3 marks]

6. The daily rainfall, R mm, in a city in Thailand during monsoon season is normally distributed with mean $\mu = 45$ mm and standard deviation $\sigma = 9$ mm.

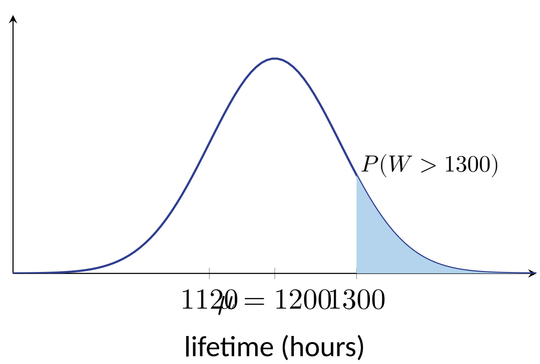
(a) Find $P(R > 50)$. **[2 marks]**

(b) A day is chosen at random. State the probability that the rainfall is less than 45 mm. **[1 marks]**

7. The length, L cm, of a species of tropical fish kept in an aquarium in Japan is normally distributed with mean $\mu = 14$ cm and standard deviation $\sigma = 2.5$ cm.
Find the value of l such that $P(L > l) = 0.15$.

[3 marks]

8. The lifetime, W hours, of a brand of light bulb sold in Germany is normally distributed with mean $\mu = 1200$ hours and standard deviation $\sigma = 80$ hours.
There are 500 light bulbs in a warehouse. Estimate the number of bulbs expected to have a lifetime greater than 1300 hours.



[3 marks]

9. The speed, V km/h, of vehicles recorded by a speed camera on a highway in Australia is normally distributed with mean $\mu = 105$ km/h and standard deviation $\sigma = 10$ km/h.
Find $P(95 < V < 115)$.

[3 marks]

10. The mass, M kg, of newborn lambs on a farm in New Zealand is normally distributed with mean $\mu = 4.2$ kg and standard deviation $\sigma = 0.6$ kg.
Find the value of m such that $P(M < m) = 0.25$.

[3 marks]

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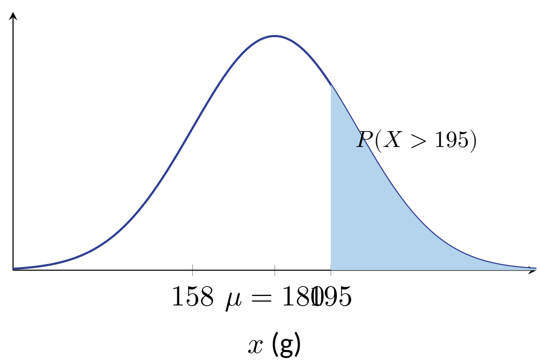
Section B — Medium

MEDIUM

11. The masses of avocados sold at a market are normally distributed with mean $\mu = 180$ g and standard deviation $\sigma = 22$ g. Let X be the mass of a randomly selected avocado, in grams.

(a) Find $P(X > 195)$. **[2 marks]**

(b) In a delivery of 400 avocados, estimate the number with a mass greater than 195 g. **[2 marks]**



12. The daily high temperature in a coastal city during summer is normally distributed with mean $27\,^\circ$C and standard deviation $3.5\,^\circ$C. Let T be the daily high temperature, in $^\circ$C.

(a) Find $P(24 < T < 32)$. **[2 marks]**

(b) Find the temperature t such that $P(T > t) = 0.10$. **[2 marks]**

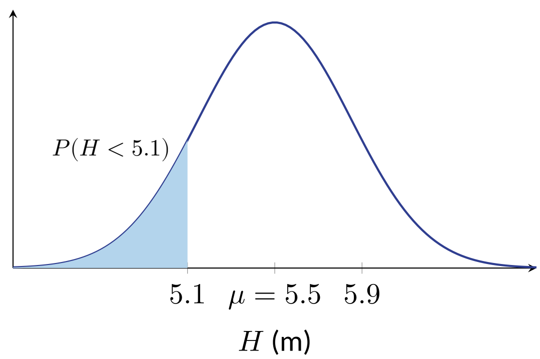
13. The time, in minutes, that customers spend waiting at a café before being served is normally distributed with mean μ and standard deviation 2.4 min. It is known that $P(\text{wait} < 3) = 0.2090$.
Find the value of μ. **[4 marks]**

14. The heights of adult male giraffes in a wildlife reserve are normally distributed with mean 5.5 m and standard deviation 0.35 m. Let H be the height of a randomly selected giraffe, in metres.

(a) Write down $P(H < 5.5)$. **[1 marks]**

(b) Find $P(H < 5.1)$. **[2 marks]**

(c) Find $P(5.1 < H < 5.9)$. **[1 marks]**



15. The scores on a standardised geography test are normally distributed with mean 62 and standard deviation σ. It is known that $15\,\%$ of students score more than 80.

(a) Find the value of σ. **[3 marks]**

(b) Find the probability that a randomly selected student scores less than 50. **[1 marks]**

16. The volumes of juice dispensed by a machine into bottles are normally distributed with mean 502 ml and standard deviation 4 ml. A bottle is rejected if it contains less than 494 ml or more than 510 ml.

(a) Find the probability that a randomly selected bottle is rejected. **[3 marks]**

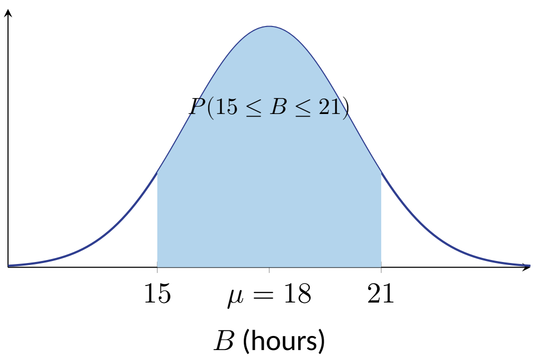
(b) Find the probability that a randomly selected bottle is accepted. **[1 marks]**

17. The lengths of fish caught in a lake are normally distributed with mean 38 cm and standard deviation 6.5 cm. Fishing regulations require that any fish shorter than L cm must be returned to the lake. The probability that a randomly caught fish must be returned is 0.05.
Find the value of L. **[4 marks]**

18. The battery life of a brand of wireless headphones is normally distributed with mean 18 hours and standard deviation 2.2 hours. Let B be the battery life of a randomly selected pair of headphones, in hours.

(a) Find $P(15 \leq B \leq 21)$. **[2 marks]**

(b) Find the battery life b such that $20\,\%$ of headphones last longer than b hours. **[2 marks]**



19. The ages of participants registered for a community marathon are normally distributed with mean 34 years and standard deviation 8 years. A participant is classified as a "junior" if they are under 22 years old, and as a "senior" if they are over 50 years old.

(a) Find the probability that a randomly selected participant is classified as a junior. **[2 marks]**

(b) There are 650 participants in total. Estimate the number who are classified as seniors. **[2 marks]**

20. The diameters of bolts produced in a factory are normally distributed with mean 12 mm and standard deviation σ mm. A bolt is defective if its diameter is less than 11 mm or greater than 13 mm. The probability that a bolt is defective is 0.0456.

(a) Find the value of σ. **[3 marks]**

(b) Find the probability that a bolt has a diameter greater than 12.5 mm. **[1 marks]**

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Section C — Hard

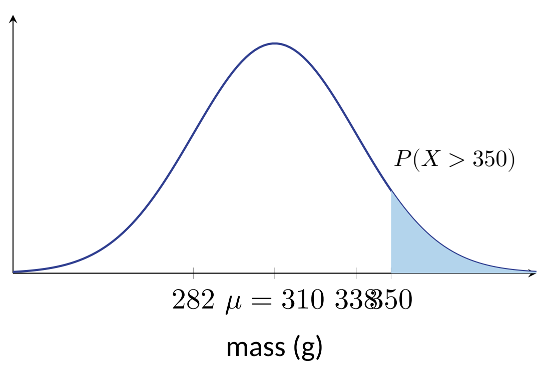
HARD

21. The masses of mangoes harvested at a farm are normally distributed with mean $\mu = 310$ g and standard deviation $\sigma = 28$ g. A mango is classified as "premium" if its mass exceeds 350 g.

(a) Find the probability that a randomly selected mango is premium. **[1 marks]**

(b) A crate contains 120 mangoes. Estimate the number of premium mangoes in the crate. **[2 marks]**

(c) The farmer changes the harvesting process so that the mean mass increases to $\mu = 325$ g while σ remains 28 g. Determine whether the expected number of premium mangoes in a crate of 120 increases by more than 10. Justify your answer. **[2 marks]**



22. The daily rental revenue X (in USD) at a city apartment follows a normal distribution with mean μ USD and standard deviation $\sigma = 45$ USD. It is known that $P(X < 180) = 0.0668$.

(a) Show that $\mu = 225$ USD. **[2 marks]**

(b) Find $P(180 < X < 270)$. **[2 marks]**

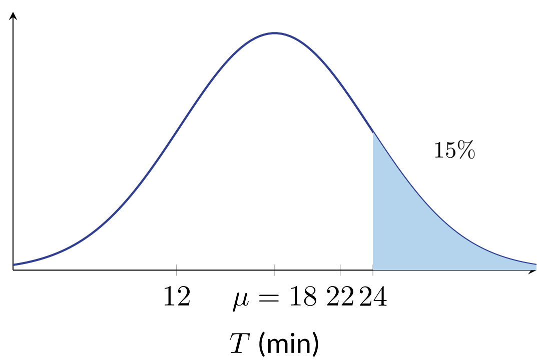
(c) The owner considers a day "exceptional" if revenue exceeds 300 USD. Find the probability that, in a randomly chosen week of 7 days, at least 2 days are exceptional. **[1 marks]**

23. The time, T minutes, taken by students to complete a logic puzzle is normally distributed with mean $\mu = 18$ min and standard deviation σ min. It is given that 15% of students take more than 24 minutes.

(a) Show that $\sigma = 5.79$ minutes (correct to 3 significant figures). **[2 marks]**

(b) Hence find $P(12 < T < 22)$. **[2 marks]**

(c) A student is chosen at random. Given that the student took more than 18 minutes, find the probability that they took more than 24 minutes. **[1 marks]**



24. A factory fills bottles of olive oil. The volume V ml in a bottle is normally distributed with mean $\mu = 502$ ml and standard deviation $\sigma = 4$ ml. A bottle is rejected if its volume is less than 494 ml or greater than 510 ml.

(a) Find the probability that a randomly selected bottle is rejected. **[2 marks]**

(b) In a production run of 2000 bottles, estimate the number of bottles that are rejected. **[1 marks]**

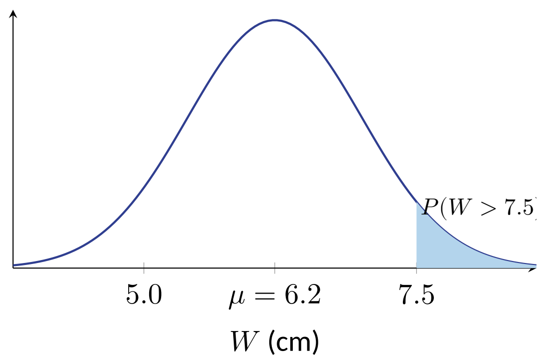
(c) The factory adjusts the machine so that μ changes but σ remains 4 ml. Find the value of μ such that only 1% of bottles have volume less than 494 ml. **[2 marks]**

25. The wingspan, W cm, of a species of butterfly is normally distributed with mean 6.2 cm and standard deviation 0.8 cm.

(a) Find $P(W > 7.5)$. **[1 marks]**

(b) A researcher collects 200 butterflies. Estimate the number with wingspan between 5.0 cm and 7.5 cm. **[2 marks]**

(c) The researcher defines a butterfly as "rare" if its wingspan places it in the top 3% of the distribution. Find the minimum wingspan for a butterfly to be classified as rare. **[2 marks]**



26. The scores, S, achieved by candidates in a professional examination are normally distributed. The pass mark is 50. It is known that 10% of candidates score below 38 and 5% score above 72.

(a) Write down two equations in μ and σ. **[1 marks]**

(b) Hence find μ and σ. **[2 marks]**

(c) Find the probability that a randomly selected candidate passes the examination. **[2 marks]**

27. A soft-drink machine dispenses liquid whose volume V ml is normally distributed with mean $\mu = 355$ ml and standard deviation $\sigma = 6$ ml. A cup overflows if it receives more than 368 ml.

(a) Find the probability that a randomly chosen cup overflows. **[1 marks]**

(b) The machine is recalibrated so that the probability of overflow is reduced to exactly 0.02. The standard deviation remains 6 ml. Find the new mean μ^*. **[2 marks]**

(c) With the original settings ($\mu = 355$, $\sigma = 6$), find the volume v such that 90% of cups receive at least v ml. **[2 marks]**

28. The concentration C (mg/L) of a medication in a patient's bloodstream t hours after administration is modelled by $C(t) = 12t\,e^{-0.5t}$ for $0 \leq t \leq 10$. Independent of this model, measurement errors mean that recorded concentrations are normally distributed about the true value with standard deviation 0.35 mg/L.
At $t = 3$ hours the true concentration is $C(3) = 12(3)e^{-1.5}$.

(a) Show that $C(3) = 8.04$ mg/L correct to 3 significant figures. **[1 marks]**

(b) A recorded reading at $t = 3$ is modelled as $R \sim N(8.04,\ 0.35^2)$. Find $P(R > 8.5)$. **[2 marks]**

(c) Find the recorded value r such that only 5% of readings exceed r. **[2 marks]**

29. A quality-control engineer tests the breaking strength B kN of steel cables. Breaking strengths follow a normal distribution with mean $\mu = 84$ kN and standard deviation $\sigma = 5$ kN. A cable fails inspection if its breaking strength is below 75 kN.

(a) Find the probability that a randomly selected cable fails inspection. **[1 marks]**

(b) A batch of 15 cables is tested. Let F be the number of cables that fail inspection. State the distribution of F. **[1 marks]**

(c) Find the probability that at most 2 cables in the batch fail inspection. **[2 marks]**

(d) Determine whether it is more likely that exactly 1 cable or exactly 2 cables fail inspection. Justify your answer. **[1 marks]**

30. The heights H cm of adult males in a country are normally distributed with mean 175 cm and standard deviation 7 cm. A doorframe is built to a height d cm so that no more than 2% of adult males need to duck.

(a) Find the minimum integer value of d such that the proportion of adult males who need to duck is at most 2%. **[3 marks]**

(b) An architect checks two independently chosen adult males. Find the probability that both must duck if $d = 189$ cm. **[2 marks]**

Worked Solutions

Q1 — Normal probability, left tail [EASY]

We require the probability that a randomly chosen apple has mass less than 195 g, which is the area to the left of 195 under the normal curve with $\mu = 180$, $\sigma = 12$. The shaded region in the diagram confirms this is a left-tail probability.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= -10^{99}$, upper $= 195$, $\mu = 180$, $\sigma = 12$ **M1**

Unrounded value: $0.89435\ldots$

$$P(X < 195) = 0.894\textbf{A2}$$

Warning: Make sure the lower bound is set to a very large negative number (e.g. -10^{99}), not zero, otherwise the GDC will give an incorrect answer.

Q2 — Normal probability, right tail [EASY]

We require the probability that waiting time exceeds 11 minutes, which is the area to the right of 11 under the normal curve with $\mu = 8$, $\sigma = 2$.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 11$, upper $= 10^{99}$, $\mu = 8$, $\sigma = 2$ **M1**

Unrounded value: $0.066807\ldots$

$$P(T > 11) = 0.0668\textbf{A2}$$

Warning: This is a right-tail probability; set the upper bound to a very large positive number (e.g. 10^{99}), not a finite value, to capture the entire right tail.

Q3 — Normal probability, central region [EASY]

We require the probability that a giraffe's height lies between 500 cm and 560 cm, which is the area between these two values under the normal curve with $\mu = 530$, $\sigma = 20$. The shaded region in the diagram confirms this is a central probability.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 500$, upper $= 560$, $\mu = 530$, $\sigma = 20$ **M1**

Unrounded value: $0.86638\ldots$

$$P(500 < H < 560) = 0.866\textbf{A2}$$

Warning: Both boundaries must be entered correctly; swapping or omitting either boundary will give the wrong region.

Q4 — Symmetry and right-tail normal [EASY]

(a) Since 250 is the mean of the distribution, exactly half the area lies below it. By symmetry of the normal distribution:

$$P(V < 250) = 0.5\textbf{A1}$$

(b) We require the probability that the dispensed volume exceeds 258 ml, the area to the right of 258.

GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 258$, upper $= 10^{99}$, $\mu = 250$, $\sigma = 5$ **M1**

Unrounded value: $0.054799\ldots$

$$P(V > 258) = 0.0548\textbf{A1}$$

Warning: Part (a) requires no GDC calculation; the answer follows directly from the symmetry property $P(X < \mu) = 0.5$ for any normal distribution.

Q5 — Inverse normal, left tail [EASY]

We need the value s below which 90% of scores fall. This is a left-tail inverse normal problem with cumulative probability 0.90.

Setting up the equation: $P(S < s) = 0.90$ with $\mu = 62$, $\sigma = 8$. **M1**

GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ invNorm, area $= 0.90$, $\mu = 62$, $\sigma = 8$, LEFT tail

Unrounded value: $72.256\ldots$

$$s = 72.3\textbf{A2}$$

Warning: The area entered in invNorm is always the area to the LEFT of the required value; since $P(S < s) = 0.90$, the tail area is already 0.90 and no subtraction from 1 is needed.

Q6 — Right-tail normal and symmetry [EASY]

(a) We require the probability that daily rainfall exceeds 50 mm, the area to the right of 50 under the normal curve with $\mu = 45$, $\sigma = 9$.

GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= 50$, upper $= 10^{99}$, $\mu = 45$, $\sigma = 9$ **M1**

Unrounded value: $0.28948\ldots$

$$P(R > 50) = 0.289\textbf{A1}$$

(b) Since 45 mm is the mean of the distribution, by symmetry exactly half the area lies below it:

$$P(R < 45) = 0.5\textbf{A1}$$

Warning: Part (b) is a symmetry result requiring no GDC; any student who attempts to use normalcdf here wastes time and risks a rounding error.

Q7 — Inverse normal, right tail [EASY]

We need the value l above which 15% of fish lengths lie. Since $P(L > l) = 0.15$, the area to the LEFT of l is $1 - 0.15 = 0.85$.

Setting up: $P(L < l) = 0.85$ with $\mu = 14$, $\sigma = 2.5$. **M1**

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.85$, $\mu = 14$, $\sigma = 2.5$, LEFT tail

Unrounded value: $16.591\ldots$

$$l = 16.6 \text{ cm} \textbf{A2}$$

Warning: The given probability is a RIGHT-tail probability; you must subtract from 1 to obtain the left-tail area before entering it into invNorm, otherwise the GDC returns the wrong boundary.

Q8 — Expected count from normal probability [EASY]

First find the probability that a single bulb has lifetime greater than 1300 hours. The shaded region in the diagram shows this is the area to the right of 1300.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 1300$, upper $= 10^{99}$, $\mu = 1200$, $\sigma = 80$ **M1**

Unrounded value: $P(W > 1300) = 0.10565\ldots$

Multiply by the total number of bulbs to estimate the expected count:

$$500 \times 0.10565\ldots = 52.82\ldots \approx 53 \text{ bulbs} \textbf{A1}$$

A1

Warning: Round the final expected count to the nearest whole number since it represents a number of physical bulbs; do not leave the answer as a decimal.

Q9 — Normal probability, central region [EASY]

We require the probability that a vehicle's recorded speed lies between 95 km/h and 115 km/h, the area between these two values under the normal curve with $\mu = 105$, $\sigma = 10$.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 95$, upper $= 115$, $\mu = 105$, $\sigma = 10$ **M1**

Unrounded value: $0.68269\ldots$

$$P(95 < V < 115) = 0.683 \textbf{A2}$$

Warning: Note that 95 and 115 are each exactly one standard deviation from the mean; the answer is the familiar 68.3% empirical rule result, but the GDC value must still be shown as evidence.

Q10 — Inverse normal, lower quartile [EASY]

We need the value m below which 25% of lamb masses lie. This is a left-tail inverse normal problem with cumulative probability 0.25.

Setting up the equation: $P(M < m) = 0.25$ with $\mu = 4.2$, $\sigma = 0.6$. **M1**

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.25$, $\mu = 4.2$, $\sigma = 0.6$, LEFT tail

Unrounded value: $3.7953\ldots$

$$m = 3.80 \text{ kg}$$ **A2**

Warning: The area 0.25 is entered directly as the left-tail area since $P(M < m) = 0.25$; no subtraction from 1 is required here.

Q11 — Normal probability and expected count [MEDIUM]

(a) To find the probability that a randomly chosen avocado has mass greater than 195 g, we use the upper-tail area of the normal distribution with $\mu = 180$, $\sigma = 22$.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 195$, upper $= 10^{99}$, $\mu = 180$, $\sigma = 22$ **M1**

$P(X > 195) = 0.24706\ldots \approx 0.247$ **A1**

(b) The expected number of avocados above 195 g is found by multiplying the probability from (a) by the total number in the delivery.

$400 \times 0.24706\ldots = 98.82\ldots \approx 99$ avocados **A1ft**

Warning: Use the unrounded probability from part (a) in this multiplication; rounding 0.247 first gives 98.8, which still rounds to 99 here, but premature rounding across parts can lose marks in other questions.

Q12 — Normal interval probability and inverse normal [MEDIUM]

(a) To find the probability that the temperature lies strictly between $24\,^\circ$C and $32\,^\circ$C, we compute the central area under the normal curve with $\mu = 27$, $\sigma = 3.5$.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 24$, upper $= 32$, $\mu = 27$, $\sigma = 3.5$ **M1**

$P(24 < T < 32) = 0.74580\ldots \approx 0.746$ **A1**

(b) We need the threshold t such that the upper-tail probability equals 0.10; this requires the inverse normal on the **right** tail, so we enter area $= 0.90$ in the left-tail inverse normal.

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.90$ (LEFT tail), $\mu = 27$, $\sigma = 3.5$ **M1**

$t = 31.485\ldots \approx 31.5\,^\circ$C **A1**

Warning: The question asks for $P(T > t) = 0.10$, which is the *upper* tail; entering area $= 0.10$ into invNorm gives the *lower* $10\,\%$ threshold ($22.5\,^\circ$C) instead — the wrong answer.

Q13 — Unknown mean from one probability condition [MEDIUM]

We are told $P(\text{wait} < 3) = 0.2090$ with $\sigma = 2.4$. We set up a normalcdf equation in the unknown μ and solve using the GDC solver.

The equation to solve is: normalcdf$(-10^{99},\ 3,\ \mu,\ 2.4) = 0.2090$ **M1**

GDC: Equation Solver (or Numerical Solver), define $f(\mu) =$ normalcdf$(-10^{99}, 3, \mu, 2.4) - 0.2090 = 0$, solve for μ **M1**

$\mu = 4.9997\ldots \approx 5.00$ min **A2**

Warning: Do not attempt to standardise algebraically and use a z-table; the GDC solver route is the required method in this course, and the printed probability 0.2090 corresponds to $z \approx -0.810$, confirming $\mu \approx 3 + 0.810 \times 2.4 \approx 5.0$ — a useful check but not the assessed method.

Q14 — Symmetry, tail probability and interval [MEDIUM]

(a) Because 5.5 m is the mean of a normal distribution, the distribution is perfectly symmetric about μ, so exactly half the area lies below the mean.

$P(H < 5.5) = 0.5$ **A1**

(b) We find the lower-tail probability below 5.1 m using the GDC.

GDC: Statistics $\rightarrow$ Distributions $\rightarrow$ normalcdf, lower $= -10^{99}$, upper $= 5.1$, $\mu = 5.5$, $\sigma = 0.35$ **M1**

$P(H < 5.1) = 0.12714\ldots \approx 0.127$ **A1**

(c) By symmetry of the normal curve about $\mu = 5.5$, the interval $(5.1, 5.9)$ is symmetric ($5.9 = 5.5 + 0.4$ and $5.1 = 5.5 - 0.4$), so

$P(5.1 < H < 5.9) = 1 - 2 \times P(H < 5.1) = 1 - 2(0.12714\ldots) = 0.74572\ldots \approx 0.746$ **A1ft**

Warning: Use the unrounded value from (b) when computing $1 - 2 \times 0.127 = 0.746$; either approach (symmetry or a fresh normalcdf) is acceptable, but the answer must be consistent with part (b).

Q15 — Unknown standard deviation from upper-tail probability [MEDIUM]

(a) We are told $P(\text{score} > 80) = 0.15$ with $\mu = 62$, and we need to find σ. We set up the normalcdf equation in σ and use the GDC solver.

The equation to solve is: normalcdf$(80,\ 10^{99},\ 62,\ \sigma) = 0.15$ **M1**

GDC: Numerical Solver, define $f(\sigma) =$ normalcdf$(80, 10^{99}, 62, \sigma) - 0.15 = 0$, solve for $\sigma > 0$ **M1**

$\sigma = 19.138\ldots \approx 19.1$ **A1**

(b) Using the value of σ found in (a), we find the probability of scoring below 50.

GDC: normalcdf$(-10^{99},\ 50,\ 62,\ 19.138\ldots) = 0.26673\ldots \approx 0.267$ **A1ft**

Warning: Use the full unrounded σ from part (a) in the GDC for part (b); rounding to 19.1 introduces a small error in the final probability.

Q16 — Rejection probability as sum of two tails [MEDIUM]

(a) A bottle is rejected if its volume falls *outside* the interval $[494, 510]$. We compute the probability of the two rejection tails separately and add them.

Lower tail: GDC: normalcdf$(-10^{99},\ 494,\ 502,\ 4) = 0.02275\ldots$ **M1**

Upper tail: GDC: normalcdf$(510,\ 10^{99},\ 502,\ 4) = 0.02275\ldots$

$P(\text{rejected}) = 0.02275\ldots + 0.02275\ldots = 0.04550\ldots \approx 0.0455$ **A1**

Alternatively: $1 -$ normalcdf$(494,\ 510,\ 502,\ 4) = 1 - 0.95449\ldots = 0.04550\ldots \approx 0.0455$ **A1**

(b) The probability of acceptance is the complement of the rejection probability.

$P(\text{accepted}) = 1 - 0.04550\ldots = 0.95449\ldots \approx 0.954$ **A1ft**

Warning: Note that $\mu = 502$ is *not* centred between 494 and 510 (494 is 8 ml below the mean while 510 is also 8 ml above), so in fact the distribution *is* symmetric here — but always verify this rather than assuming it from the limits.

Q17 — Inverse normal lower tail threshold [MEDIUM]

We need the length L such that $P(H < L) = 0.05$, where $H \sim N(38,\ 6.5^2)$. This is a lower-tail inverse normal problem.

We require the value at the **left** tail with area 0.05. **M1**

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.05$ (LEFT tail), $\mu = 38$, $\sigma = 6.5$ **M1**

$L = 27.308\ldots \approx 27.3$ cm **A2**

Warning: The $5\,\%$ of fish that must be returned are the *shortest* fish, so the threshold is in the **left** tail. Entering area $= 0.95$ in invNorm gives the right-tail threshold (48.7 cm), which is the wrong end of the distribution.

Q18 — Normal interval and inverse normal upper tail [MEDIUM]

(a) To find the probability that battery life falls between 15 and 21 hours, we compute the central area for $B \sim N(18,\ 2.2^2)$.

GDC: Statistics $\to$ Distributions $\to$ normalcdf, lower $= 15$, upper $= 21$, $\mu = 18$, $\sigma = 2.2$ **M1**

$P(15 \le B \le 21) = 0.83066\ldots \approx 0.831$ **A1**

(b) We need b such that $P(B > b) = 0.20$; this is an upper-tail condition, so the left-tail area entering invNorm is $1 - 0.20 = 0.80$.

GDC: Statistics $\to$ Distributions $\to$ invNorm, area $= 0.80$ (LEFT tail), $\mu = 18$, $\sigma = 2.2$ **M1**

$b = 19.850\ldots \approx 19.9$ hours **A1**

Warning: The question states that $20\,\%$ of headphones last *longer* than b, meaning b is in the upper portion of the distribution. Entering area $= 0.20$ into invNorm gives the *lower* $20\,\%$ threshold (16.2 hours) — the wrong answer.

Q19 — Two-tail probabilities and expected count [MEDIUM]

(a) A junior participant is under 22 years old. We find the lower-tail probability for $A \sim N(34,\ 8^2)$.

GDC: Statistics → Distributions → normalcdf, lower $= -10^{99}$, upper $= 22$, $\mu = 34$, $\sigma = 8$ **M1**

$P(A < 22) = 0.06681\ldots \approx 0.0668$ **A1**

(b) A senior is over 50 years old. We first find $P(A > 50)$, then multiply by the total number of participants to estimate the count.

GDC: normalcdf$(50,\ 10^{99},\ 34,\ 8) = 0.02275\ldots$

Expected number of seniors $= 650 \times 0.02275\ldots = 14.788\ldots \approx 15$ participants **A2**

Warning: The two age-group boundaries (22 and 50) are not symmetric about $\mu = 34$ ($22 = 34 - 12$, $50 = 34 + 16$), so the two probabilities are *not* equal — do not assume symmetry and copy the answer from (a).

Q20 — Unknown standard deviation from two-tail probability [MEDIUM]

(a) The defect condition is $|d - 12| > 1$, i.e. diameter outside $(11, 13)$. Because $\mu = 12$ is centred in the interval, both tails are equal, so $P(\text{defective}) = P(D < 11) + P(D > 13) = 0.0456$.

By symmetry, $P(D < 11) = 0.0228$. We set up the equation normalcdf$(-10^{99},\ 11,\ 12,\ \sigma) = 0.0228$ and solve for σ. **M1**

Equivalently, normalcdf$(-10^{99},\ 11,\ 12,\ \sigma) = 0.0456/2 = 0.0228$.

GDC: Numerical Solver, $f(\sigma) =$ normalcdf$(-10^{99}, 11, 12, \sigma) - 0.0228 = 0$, solve for $\sigma > 0$ **M1**

$\sigma = 0.49913\ldots \approx 0.499$ mm **A1**

(b) Using $\mu = 12$ and $\sigma = 0.49913\ldots$, we find the upper-tail probability above 12.5 mm.

GDC: normalcdf$(12.5,\ 10^{99},\ 12,\ 0.49913\ldots) = 0.15756\ldots \approx 0.158$ **A1ft**

Warning: The symmetry argument ($P(D < 11) = P(D > 13) = 0.0228$) only holds because $\mu = 12$ is exactly midway between 11 and 13; if the mean were not centred, you would need to solve the full two-tail equation without invoking symmetry.

Q21 — Premium mango proportion and expected count [HARD]

(a) We need the probability that a mango's mass exceeds 350 g, so we evaluate $P(X > 350)$ where $X \sim N(310, 28^2)$.

GDC: Statistics → Distributions → Normal CDF, lower $= 350$, upper $= 10^{99}$, $\mu = 310$, $\sigma = 28$.

$P(X > 350) = 0.07656\ldots \approx 0.0766$ **A1**

(b) The expected number of premium mangoes is found by multiplying the sample size by the probability from part (a).

$(M1)$: Expected number $= 120 \times 0.07656\ldots$ **(M1)**

$= 9.187\ldots \approx 9.19$ mangoes **A1ft**

(c) We repeat the calculation for the new mean $\mu = 325$ g and compare the increase with 10.

GDC: Normal CDF, lower $= 350$, upper $= 10^{99}$, $\mu = 325$, $\sigma = 28$: $P(X > 350) = 0.18624\ldots$

New expected number $= 120 \times 0.18624\ldots = 22.35\ldots$

Increase $= 22.35\ldots - 9.19\ldots = 13.16\ldots > 10$ **R1**

Therefore the expected number of premium mangoes does increase by more than 10. **A1**

Warning: Using the rounded value 0.0766 instead of the unrounded $0.07656\ldots$ in part (b) and then carrying that rounded figure into part (c) will accumulate error and may give an incorrect conclusion at the comparison step.

Q22 — Apartment revenue: reversal then fusion [HARD]

(a) We are told $P(X < 180) = 0.0668$ with $\sigma = 45$; we must recover μ and verify it equals 225.

Setting up the equation: $P(X < 180) = 0.0668$ with $X \sim N(\mu, 45^2)$.

GDC: Statistics $\to$ Distributions $\to$ Inverse Normal, area $=$ 0.0668, LEFT tail, $\mu = 0$, $\sigma = 1$ gives $z = -1.4998\ldots \approx -1.500$.

Standardising: $\dfrac{180 - \mu}{45} = -1.500$, so $180 - \mu = -67.49\ldots$, giving $\mu = 247\ldots$

Alternative (solver route): GDC solver on normalcdf$(-10^{99}, 180, \mu, 45) = 0.0668$ gives $\mu = 225.0$ **M1**

$\mu = 225$ USD **AG**

(b) With $X \sim N(225, 45^2)$ we find $P(180 < X < 270)$.

GDC: Normal CDF, lower $= 180$, upper $= 270$, $\mu = 225$, $\sigma = 45$. **M1**

$P(180 < X < 270) = 0.68269\ldots \approx 0.683$ **A1**

(c) First find $p = P(X > 300)$: GDC Normal CDF, lower $= 300$, upper $= 10^{99}$, $\mu = 225$, $\sigma = 45$ gives $p = 0.04779\ldots$.

Let $Y \sim B(7, 0.04779\ldots)$. $P(Y \geq 2) = 1 - P(Y \leq 1)$.

GDC: Binomial CDF, $n = 7$, $p = 0.04779\ldots$, $x = 1$: $P(Y \leq 1) = 0.93987\ldots$

$P(Y \geq 2) = 1 - 0.93987\ldots = 0.06013\ldots \approx 0.0601$ **A1**

Warning: The tail direction in the inverse-normal step of part (a) is LEFT (smallest values), not RIGHT; reversing the tail gives $\mu = 135$ which is far from the printed target.

Q23 — Logic puzzle times: find sigma then conditional probability [HARD]

(a) We know $P(T > 24) = 0.15$ with $\mu = 18$; we must find σ and verify $\sigma = 5.79$ min.

GDC solver: solve normalcdf$(24, 10^{99}, 18, \sigma) = 0.15$ for σ. **M1**

Unrounded value: $\sigma = 5.7918\ldots$ min

$\sigma = 5.79$ min (3 s.f.) **AG**

(b) Using $T \sim N(18, 5.79^2)$ (unrounded $5.7918\ldots$), we find $P(12 < T < 22)$.

GDC: Normal CDF, lower $= 12$, upper $= 22$, $\mu = 18$, $\sigma = 5.7918\ldots$. **M1**

$P(12 < T < 22) = 0.65126\ldots \approx 0.651$ **A1**

(c) We want $P(T > 24 \mid T > 18)$. Since the normal distribution is symmetric about the mean, $P(T > 18) = 0.5$.

$$P(T > 24 \mid T > 18) = \frac{P(T > 24 \cap T > 18)}{P(T > 18)} = \frac{P(T > 24)}{0.5} = \frac{0.15}{0.5} = 0.30$$ **A1**

Warning: Using the rounded $\sigma = 5.79$ in part (b) instead of the full unrounded solver value $5.7918\ldots$ will give a slightly different (penalisable) answer if the rounding propagates.

Q24 — Olive oil bottles: two-tail rejection and reversal [HARD]

(a) We need $P(V < 494) + P(V > 510)$ with $V \sim N(502, 4^2)$.

GDC: Normal CDF for left tail: lower $= -10^{99}$, upper $= 494$, $\mu = 502$, $\sigma = 4$: $P(V < 494) = 0.02275\ldots$ **(M1)**

GDC: Normal CDF for right tail: lower $= 510$, upper $= 10^{99}$: $P(V > 510) = 0.02275\ldots$

$P(\text{rejected}) = 0.02275\ldots + 0.02275\ldots = 0.04550\ldots \approx 0.0455$ **A1**

(b) Expected number rejected $= 2000 \times 0.04550\ldots = 91.0\ldots \approx 91$ bottles. **A1**

(c) We now require $P(V < 494) = 0.01$ with $\sigma = 4$ and unknown μ^*.

GDC solver: normalcdf$(-10^{99}, 494, \mu^*, 4) = 0.01$. **M1**

$\mu^* = 494 - 4 \times \text{invNorm}(0.01) = 494 + 4 \times 2.3263\ldots = 503.305\ldots$

$\mu^* = 503$ ml (3 s.f.) **A1**

Warning: This question uses a TWO-sided rejection region in part (a); adding only one tail gives half the correct rejection probability. In part (c), the left-tail condition means the mean must be *above* 494 ml — mistaking the tail direction gives $\mu^* \approx 484.7$ ml.

Q25 — Butterfly wingspan: proportion, count, and top-3% threshold [HARD]

(a) With $W \sim N(6.2, 0.8^2)$, evaluate $P(W > 7.5)$.

GDC: Normal CDF, lower $= 7.5$, upper $= 10^{99}$, $\mu = 6.2$, $\sigma = 0.8$.

$P(W > 7.5) = 0.05208\ldots \approx 0.0521$ **A1**

(b) First find $P(5.0 < W < 7.5)$: GDC Normal CDF, lower $= 5.0$, upper $= 7.5$, $\mu = 6.2$, $\sigma = 0.8$. **M1**

$P(5.0 < W < 7.5) = 0.89134\ldots$

Expected count $= 200 \times 0.89134\ldots = 178.27\ldots \approx 178$ butterflies. **A1**

(c) The top 3% means $P(W > w) = 0.03$, so we need the 97th percentile, which is the RIGHT tail — the inverse-normal input is area to the LEFT $= 0.97$.

GDC: Statistics $\to$ Distributions $\to$ Inverse Normal, area $= 0.97$, $\mu = 6.2$, $\sigma = 0.8$, LEFT tail. **M1**

$w = 7.7042\ldots \approx 7.70$ cm **A1**

Warning: "Top 3%" is the RIGHT tail, but inverse-normal requires the LEFT-tail cumulative area $= 0.97$, not 0.03. Entering 0.03 gives the *bottom* 3% threshold (≈ 4.70 cm).

Q26 — Exam scores: two-condition parameter hunt [HARD]

(a) Let $S \sim N(\mu, \sigma^2)$. The two probability conditions translate directly to:

$P(S < 38) = 0.10$ and $P(S > 72) = 0.05$ **A1**

(b) Converting each condition to a z-score equation using GDC inverse-normal:

From $P(S < 38) = 0.10$: GDC invNorm, area $= 0.10$, LEFT tail gives $z_1 = -1.2816\ldots$, so $\dfrac{38 - \mu}{\sigma} = -1.2816\ldots$ **M1**

From $P(S > 72) = 0.05$: GDC invNorm, area $= 0.95$, LEFT tail gives $z_2 = 1.6449\ldots$, so $\dfrac{72 - \mu}{\sigma} = 1.6449\ldots$

Solving simultaneously: subtracting gives $\sigma(1.2816 + 1.6449) = 34$, $\sigma = \dfrac{34}{2.9265\ldots} = 11.62\ldots \approx 11.6$.

Then $\mu = 38 + 1.2816 \times 11.62\ldots = 52.89\ldots \approx 52.9$.

$\mu = 52.9$, $\sigma = 11.6$ (3 s.f.) **A1**

(c) With $S \sim N(52.9, 11.6^2)$ (use unrounded values), find $P(S > 50)$.

GDC: Normal CDF, lower $= 50$, upper $= 10^{99}$, $\mu = 52.89\ldots$, $\sigma = 11.62\ldots$ **M1**

$P(S > 50) = 0.5987\ldots \approx 0.599$ **A1**

Warning: Use the full unrounded μ and σ throughout; rounding to $\mu = 52.9$ and $\sigma = 11.6$ before part (c) causes a follow-through error in the final probability.

Q27 — Drinks machine: overflow probability, reversal, and lower percentile [HARD]

(a) With $V \sim N(355, 6^2)$, find $P(V > 368)$.

GDC: Normal CDF, lower $= 368$, upper $= 10^{99}$, $\mu = 355$, $\sigma = 6$.

$P(V > 368) = 0.01500\ldots \approx 0.0150$ **A1**

(b) We require $P(V > 368) = 0.02$ with $\sigma = 6$ and unknown μ^*; this is a rightward-tail reversal.

GDC solver: normalcdf$(368, 10^{99}, \mu^*, 6) = 0.02$. **M1**

Equivalently, 368 is the 98th percentile of $N(\mu^*, 36)$: $\mu^* = 368 - 6 \times$ invNorm$(0.98) = 368 - 6 \times 2.0537\ldots = 355.68\ldots$

$\mu^* = 355.68\ldots \approx 356$ ml (3 s.f.) **A1**

(c) We want v such that $P(V \geq v) = 0.90$, i.e. $P(V < v) = 0.10$; this is the LEFT tail at the 10th percentile.

GDC: Inverse Normal, area $= 0.10$, $\mu = 355$, $\sigma = 6$, LEFT tail. **M1**

$v = 347.31\ldots \approx 347$ ml **A1**

Warning: In part (c), "at least v ml" means the LEFT tail gives $P(V < v) = 0.10$, not $P(V < v) = 0.90$. Entering area $= 0.90$ gives the 90th percentile (≈ 362.7 ml), which answers the wrong question.

Q28 — Medication concentration: show that, normal probability, and inverse [HARD]

(a) Evaluate $C(3) = 12(3)e^{-0.5\times3} = 36e^{-1.5}$.

$36e^{-1.5} = 36 \times 0.22313\ldots = 8.0327\ldots$ **M1**

$= 8.04$ mg/L (3 s.f.) **AG**

(b) With $R \sim N(8.04, 0.35^2)$, find $P(R > 8.5)$.

GDC: Normal CDF, lower $= 8.5$, upper $= 10^{99}$, $\mu = 8.04$, $\sigma = 0.35$. **M1**

$P(R > 8.5) = 0.09416\ldots \approx 0.0942$ **A1**

(c) We want r such that $P(R > r) = 0.05$, i.e. the 95th percentile; the LEFT-tail cumulative area is 0.95.

GDC: Inverse Normal, area $= 0.95$, $\mu = 8.04$, $\sigma = 0.35$, LEFT tail. **M1**

$r = 8.04 + 0.35 \times 1.6449\ldots = 8.6157\ldots \approx 8.62$ mg/L **A1**

Warning: "Only 5% of readings *exceed* r" places r in the RIGHT tail; the inverse-normal area input must be 0.95 (LEFT cumulative), not 0.05. Using 0.05 gives the 5th percentile (≈ 7.46 mg/L), the wrong end of the distribution.

Q29 — Steel cables: normal to binomial fusion with justified decision [HARD]

(a) With $B \sim N(84, 5^2)$, find $P(B < 75)$.

GDC: Normal CDF, lower $= -10^{99}$, upper $= 75$, $\mu = 84$, $\sigma = 5$.

$P(B < 75) = 0.03593\ldots \approx 0.0359$ **A1**

(b) Since each cable is tested independently and the probability of failure is constant, $F \sim B(15,\ 0.03593\ldots)$. **A1**

(c) Find $P(F \leq 2) = P(F = 0) + P(F = 1) + P(F = 2)$.

GDC: Binomial CDF, $n = 15$, $p = 0.03593\ldots$, upper $= 2$. **M1**

$P(F \leq 2) = 0.99053\ldots \approx 0.991$ **A1**

(d) Compute both individual probabilities:

GDC Binomial PDF: $P(F = 1) = 0.37736\ldots$ and $P(F = 2) = 0.20018\ldots$

Since $P(F = 1) = 0.377 > P(F = 2) = 0.200$, exactly 1 cable failing is more likely. **R1**

Warning: The probability p for the binomial in part (b) must be the *unrounded* value from part (a); rounding p to 0.036 before the CDF/PDF calculations introduces errors at the third significant figure of the binomial probabilities.

Q30 — Doorframe height: integer bracketing and combined probability [HARD]

(a) We need the smallest integer d such that $P(H > d) \leq 0.02$, i.e. $P(H \leq d) \geq 0.98$, where $H \sim N(175, 7^2)$.

GDC: Inverse Normal, area $= 0.98$, $\mu = 175$, $\sigma = 7$, LEFT tail gives the continuous threshold: **M1**

$d^* = 175 + 7 \times 2.0537\ldots = 189.376\ldots$ cm **A1**

Since d must be an integer, we check both candidates:

$d = 189$: $P(H > 189) =$ Normal CDF$(189, 10^{99}, 175, 7) = 0.02275\ldots > 0.02$ (fails condition).

$d = 190$: $P(H > 190) =$ Normal CDF$(190, 10^{99}, 175, 7) = 0.01606\ldots \leq 0.02$ (satisfies condition). **A1**

Minimum integer value of d is **190** cm.

(b) With $d = 189$ cm, $P(H > 189) = 0.02275\ldots$ (from above). The two males are chosen independently, so:

$P(\text{both duck}) = (0.02275\ldots)^2 = 0.000518\ldots \approx 0.000518$ **M1**

$= 5.18 \times 10^{-4}$ **A1**

Warning: The integer-bracketing check in part (a) is mandatory; simply rounding 189.376 down to 189 without verification gives the wrong answer because $d = 189$ does *not* satisfy $P(H > d) \leq 0.02$.