IB Math AI SL — Topic 5
Differentiation

Derivatives & Powers of x · Tangents & Normals · Stationary Points · Optimisation Models

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

Key Formulae

- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$ (valid for all $n \in \mathbb{R}$)
- **Constant and Linear Rules:** $\frac{d}{dx}(c) = 0$; $\frac{d}{dx}(ax+b) = a$
- **Rewriting before differentiating:** $\frac{a}{x^n} = ax^{-n}$, so $\frac{d}{dx}(ax^{-n}) = -nax^{-n-1}$
- **Gradient of tangent at $x = a$:** $m_{\text{tan}} = f'(a)$; equation: $y - f(a) = f'(a)(x-a)$
- **Gradient of normal at $x = a$:** $m_{\text{norm}} = -\frac{1}{f'(a)}$; equation: $y - f(a) = -\frac{1}{f'(a)}(x-a)$
- **Increasing / Decreasing:** $f'(x) > 0 \Rightarrow$ increasing; $f'(x) < 0 \Rightarrow$ decreasing
- **Stationary points:** solve $f'(x) = 0$; classify by sign of f' either side (sign table or GDC graph)

GDC Tips

- **Evaluating the derivative at a point:** Enter $f(x)$ in Graph, then use Analysis → Sketch → Tangent at the desired x-value; the gradient is displayed, or use Calc → dy/dx and type the x-value.
- **Finding stationary points (solve $f'(x) = 0$):** Enter $f'(x)$ in Equation/Graph, then Analysis → G-Solve → Root to read each zero of the derivative directly.
- **Classifying a stationary point:** Graph $f(x)$, zoom around the stationary point, then use G-Solve → Maximum or G-Solve → Minimum to confirm the type and read the coordinates.
- **Tangent/Normal line equations:** In Graph, enter $f(x)$, go to Analysis → Sketch → Tangent (or Normal), input the x-value; the GDC draws the line and shows its equation.
- **Finding the x-value where the gradient equals a given value k:** Enter $Y_1 = f'(x)$ and $Y_2 = k$, then Analysis → G-Solve → Intersect to find x.
- **Checking increasing/decreasing intervals:** Graph $f'(x)$ and inspect where it lies above ($f' > 0$) or below ($f' < 0$) the x-axis using G-Solve → Root for boundary points.

Common Mistakes to Avoid

1. **Sign error on negative powers.** When differentiating $\frac{a}{x^2} = ax^{-2}$, the power rule gives $-2ax^{-3}$, not $+2ax^{-3}$. Always rewrite the term explicitly as ax^{-n} before applying the rule, and double-check the sign of $n \cdot a$ when n is negative.
2. **Forgetting to rewrite before differentiating.** You cannot apply the power rule to $\frac{3}{x^2}$ in fraction form. Rewrite as $3x^{-2}$ first — this step must be shown in your working or method marks are lost.
3. **Using the tangent gradient for the normal.** The normal at a point is perpendicular to the tangent, so its gradient is $-\frac{1}{f'(a)}$. Failing to take the negative reciprocal and instead using $f'(a)$ directly gives an equation for the tangent, not the normal.
4. **Asserting a maximum or minimum without evidence.** Stating "it is a minimum" earns no marks. You must provide a sign table of f' on both sides of the stationary point, or describe the GDC graph showing the function changes from decreasing to increasing (or vice versa).
5. **Premature rounding in chained calculations.** If you round $f'(a)$ to 3 s.f. before using it in the tangent or normal equation, the final equation will be inaccurate and follow-through marks may not recover the loss. Keep unrounded values until the last step, then round.
6. **Misidentifying increasing/decreasing intervals from $f'(x) = 0$ alone.** The boundary of an increasing interval is where $f'(x) = 0$, but you must check the sign of f' on each side. A stationary point that is a point of inflection does not change the increasing/decreasing status, so listing it as a boundary of an interval is incorrect.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator [3 marks]

The height of water in a fountain, h metres, is modelled by

$$f(x) = 3x^2 - 4x + 1,$$

where x is the horizontal distance in metres from the base.

(a) Write down the value of $f(0)$. [1]

(b) Find $f'(x)$. [2]

2. EASY Calculator [3 marks]

A jogger's distance from a starting point, d metres, is modelled by

$$g(t) = 5t^2 + 2t,$$

where t is the time in seconds.
Find $g'(t)$ and hence find the rate of change of distance at $t = 3$ seconds.

3. **EASY** **Calculator** **[3 marks]**

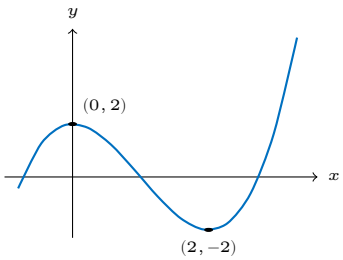
The temperature of a cooling drink, T degrees Celsius, is given by

$$T(x) = 20 - 6x + x^2,$$

where x is the time in minutes after being removed from a fridge.
Find the gradient of the curve at $x = 4$.

4. **EASY** **Calculator** **[3 marks]**

A curve has equation $y = x^3 - 3x^2 + 2$. The diagram below shows the curve with its stationary points labelled.



(a) Find $\frac{dy}{dx}$. [2]

(b) Write down the x-coordinates of the stationary points. [1]

5. **EASY** **Calculator** **[3 marks]**

The profit, P thousand dollars, made by a small bakery is modelled by

$$P(n) = -2n^2 + 12n - 10,$$

where n is the number of batches of bread baked per day.
Find the value of n that maximises the profit.

6. EASY Calculator **[3 marks]**

A curve is defined by $y = 4x^3 - x$. Find the equation of the tangent to the curve at the point where $x = 1$.

7. EASY Calculator **[3 marks]**

The depth of water in a harbour, D metres, at time t hours after midnight is modelled by

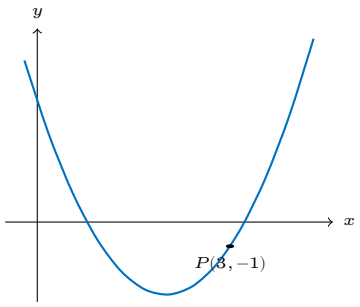
$$D(t) = t^2 - 8t + 20, \qquad 0 \leq t \leq 8.$$

(a) Find $D'(t)$. **[1]**

(b) Find the interval of t for which the depth is decreasing. **[2]**

8. EASY Calculator **[3 marks]**

The curve $y = 2x^2 - 8x + 5$ is shown below, with point P labelled.



Find the equation of the normal to the curve at $P(3, -1)$.

9. **EASY** **Calculator** **[3 marks]**

A function is given by $f(x) = x^3 - 6x^2 + 9x + 1$.

(a) Find $f'(x)$. [1]

(b) Determine the interval on which f is increasing. [2]

10. **EASY** **Calculator** **[3 marks]**

A particle moves along a straight track. Its displacement, s metres, from a fixed point is given by

$$s(t) = 3t^2 - 6t + 4,$$

where t is the time in seconds, $t \geq 0$.
Find the time at which the particle is momentarily at rest (i.e. has zero velocity).

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator [4 marks]

The depth of water in a harbour, d metres, at time t hours after midnight is modelled by

$$d(t) = 0.5t^3 - 6t^2 + 18t + 2, \qquad 0 \leq t \leq 8.$$

(a) Find $d'(t)$. [2]

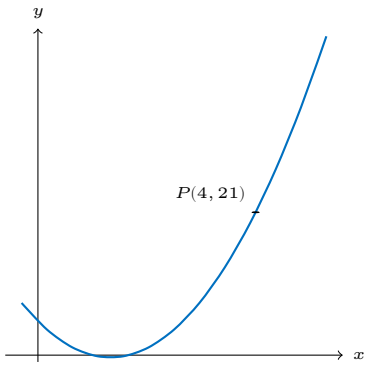
(b) Find the rate of change of depth at $t = 3$. State the units of your answer. [2]

12. MEDIUM Calculator [4 marks]

A curve has equation $f(x) = 3x^2 - 8x + 5$.

(a) Find the gradient of the curve at the point where $x = 4$. [2]

(b) Find the equation of the tangent to the curve at $x = 4$. Give your answer in the form $y = mx + c$. [2]



13. MEDIUM Calculator [4 marks]

A function is defined by $f(x) = x^3 - 6x^2 + 9x + 1$.

(a) Find the values of x at which f has stationary points. [2]

(b) Determine whether each stationary point is a local maximum or a local minimum. Justify your answer. [2]

14. MEDIUM Calculator [4 marks]

The profit, P thousand dollars, made by a small business selling x hundred units of a product is modelled by

$$P(x) = -2x^2 + 12x - 7, \qquad 0 \leq x \leq 6.$$

(a) Find the value of x that maximises the profit. [2]

(b) Find the maximum profit. [2]

15. MEDIUM Calculator [4 marks]

A curve is given by $g(x) = \dfrac{4}{x^2} + 3x$, $x \neq 0$.

(a) Rewrite $g(x)$ using a negative power of x and hence find $g'(x)$. [2]

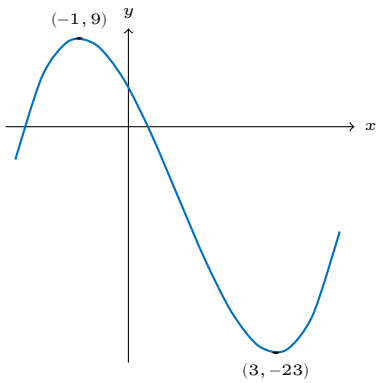
(b) Find the gradient of g at $x = 2$. [2]

16. MEDIUM Calculator [4 marks]

A function is given by $h(x) = x^3 - 3x^2 - 9x + 4$.

(a) Find the interval on which h is decreasing. [2]

(b) Find the coordinates of the local minimum of h. [2]



17. MEDIUM Calculator [4 marks]

The curve C has equation $f(x) = 2x^2 - 5x + 1$.

(a) Find the equation of the normal to C at the point where $x = 3$. [3]

(b) Write down the gradient of the tangent to C at $x = 3$. [1]

18. MEDIUM Calculator [4 marks]

The height, h metres, of a ball t seconds after it is launched is given by

$$h(t) = -4.9t^2 + 14.7t + 1.2, \qquad t \geq 0.$$

(a) Find $h'(t)$. [1]

(b) Find the time at which the ball reaches its maximum height. [2]

(c) State what $h'(t) < 0$ means in this context. [1]

19. MEDIUM Calculator [4 marks]

A curve has equation $f(x) = x^3 - 12x + 2$.

(a) Find the x-coordinates of the points on the curve where the gradient equals 0. [1]

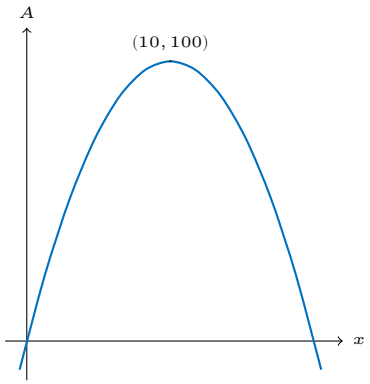
(b) Find the x-coordinates of the points on the curve where the gradient equals 24. [3]

20. MEDIUM Calculator [4 marks]

A rectangular garden has a fixed perimeter of 40 m. Let x metres be the width of the garden.

(a) Show that the area of the garden can be written as $A(x) = 20x - x^2$. [1]

(b) Find the value of x that gives the maximum area, and state the maximum area. [3]



Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator [5 marks]

The profit, P (thousands of dollars), earned by a market stall selling handmade candles depends on the price per candle, x (dollars), where $2 \leq x \leq 20$. The profit function is given by

$$P(x) = -2x^3 + 30x^2 - 100x + 80.$$

(a) Find $P'(x)$. [1]

(b) Find the value of x that maximises the profit. Justify your answer. [3]

(c) State the maximum profit, giving your answer in dollars. [1]

22. HARD Calculator [5 marks]

A function is defined as $f(x) = 3x^2 - \dfrac{48}{x^2}$ for $x > 0$.

(a) Show that $f'(x) = 6x + \dfrac{96}{x^3}$. [2]

(b) Determine whether f is increasing or decreasing for all $x > 0$. Justify your answer. [2]

(c) Find the equation of the tangent to f at the point where $x = 2$. [1]

23. HARD Calculator **[5 marks]**

A rectangular swimming pool is to be built with one side along a garden wall. Fencing is needed only for the three remaining sides. The total length of fencing available is 60 metres. Let x metres be the length of each of the two equal sides perpendicular to the wall, and let A m^2 be the area of the pool.

(a) Show that $A = 60x - 2x^2$. [1]

(b) Find $\dfrac{dA}{dx}$. [1]

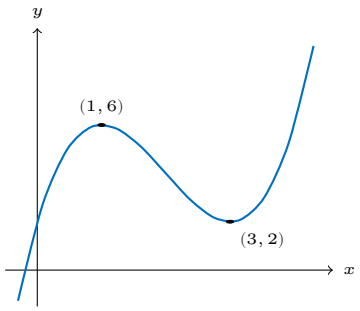
(c) Find the value of x that gives the maximum area. [1]

(d) Hence find the maximum area of the pool. [1]

(e) Explain why the model $A = 60x - 2x^2$ is not appropriate when $x = 35$. [1]

24. HARD Calculator **[5 marks]**

Let $g(x) = x^3 - 6x^2 + 9x + 2$.



(a) Write down the coordinates of the local maximum and local minimum of g. [1]

(b) Find the equation of the normal to g at the point where $x = 2$. [3]

(c) Find the values of x for which g is increasing. [1]

25. HARD Calculator **[5 marks]**

The temperature of a metal rod, $T\,^{\circ}\text{C}$, is modelled by

$$T(t) = \frac{120}{t} + 4t - 10, \qquad t > 0,$$

where t is the time in minutes after heating begins.

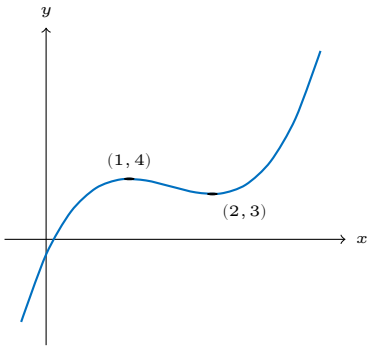
(a) Rewrite $T(t)$ using a negative power of t before differentiating. Hence find $T'(t)$. [2]

(b) Find the time t at which the temperature is at a minimum. Justify your answer. [2]

(c) Find the minimum temperature of the rod. [1]

26. HARD Calculator **[5 marks]**

Let $h(x) = 2x^3 - 9x^2 + 12x - 1$.



(a) Find $h'(x)$. [1]

(b) Find the gradient of h at $x = 3$. [1]

(c) Find the equation of the tangent to h at $x = 3$. [2]

(d) Determine the value(s) of x where the gradient of h equals 0. [1]

27. HARD Calculator **[5 marks]**

A drone flies along a straight path. Its height, H metres, above the ground is modelled by

$$H(t) = -t^3 + 9t^2 - 15t + 30, \qquad 0 \leq t \leq 8,$$

where t is the time in seconds after the drone is launched.

(a) Find $H'(t)$. [1]

(b) Find the times at which the drone is at a stationary point in its flight. [1]

(c) Determine which stationary point is a local maximum and which is a local minimum. Justify your answer using a sign diagram of $H'(t)$. [2]

(d) Find the maximum height of the drone in the interval $0 \leq t \leq 8$. [1]

28. HARD Calculator **[5 marks]**

A function is given by $f(x) = ax^2 + bx + 3$, where a and b are constants.
It is known that $f'(1) = 0$ and $f(1) = 7$.

(a) Show that $a + b = 0$ and $a + b = 4$. Identify the error in this approach and instead form two correct equations in a and b. [2]

(b) Solve the two correct equations to find a and b. [2]

(c) Write down the coordinates of the stationary point of f. [1]

29. HARD Calculator **[5 marks]**

The curve C has equation $y = x^3 - 3x^2 - 9x + 5$.

(a) Find $\frac{dy}{dx}$. [1]

(b) Find the coordinates of the two stationary points on C. [2]

(c) The point $P(-1, 10)$ lies on C. Find the equation of the normal to C at P. [2]

30. HARD Calculator **[5 marks]**

A container is made in the shape of a closed cylinder. The total surface area of the cylinder is 300π cm^2. The radius of the cylinder is r cm and the height is h cm.

(a) Show that the volume of the cylinder can be written as

$$V = 150\pi r - \pi r^3.$$

[2]

(b) Find $\frac{dV}{dr}$. [1]

(c) Find the value of r that maximises the volume. Justify your answer. [2]

Worked Solutions

Q1 — Differentiating a Quadratic Height Model EASY

(a) Substitute $x = 0$ directly into $f(x)$: $f(0) = 3(0)^2 - 4(0) + 1 = 1$ **A1**

(b) Differentiate each term using the power rule: $f'(x) = 6x - 4$ **A1A1**

Note: The constant term $+1$ vanishes on differentiation; a common error is to write $f'(x) = 6x - 4 + 1$.

Q2 — Rate of Change of Distance EASY

Differentiate $g(t) = 5t^2 + 2t$ term by term: $g'(t) = 10t + 2$ **M1A1**

Substitute $t = 3$: $g'(3) = 10(3) + 2 = 32$ m s^{-1} **A1**

Note: The answer must include units (m s^{-1}) to fully interpret the rate of change in context.

Q3 — Gradient of a Cooling Curve EASY

Differentiate $T(x) = 20 - 6x + x^2$ term by term: $T'(x) = 2x - 6$ **M1A1**

Substitute $x = 4$: $T'(4) = 2(4) - 6 = 2$ **A1**

Note: The gradient here is positive, meaning the temperature is increasing at $x = 4$; do not confuse this with the drink still cooling.

Q4 — Stationary Points of a Cubic EASY

(a) Differentiate $y = x^3 - 3x^2 + 2$ term by term: $\dfrac{dy}{dx} = 3x^2 - 6x$ **A1A1**

(b) At stationary points $\dfrac{dy}{dx} = 0$: factorise $3x(x - 2) = 0$, confirmed by the labelled diagram.

GDC: Graph $y = 3x^2 - 6x$ and find zeros. $x = 0$ and $x = 2$ **A1**

Note: The question asks only for the x-coordinates; writing the full coordinate pairs is acceptable but writing only the y-values would not earn the mark.

Q5 — Maximising Profit EASY

Differentiate $P(n) = -2n^2 + 12n - 10$: $P'(n) = -4n + 12$ **M1**

Set $P'(n) = 0$: $-4n + 12 = 0 \Rightarrow n = 3$ **A1**

Since $P'(n) > 0$ for $n < 3$ and $P'(n) < 0$ for $n > 3$, the function changes from increasing to decreasing, confirming $n = 3$ gives a local maximum. The profit is maximised when $n = 3$ batches are baked per day.

A1

Note: Always verify the nature of the stationary point using a sign table of P'; simply finding where $P' = 0$ without classification is incomplete.

Q6 — Tangent to a Cubic Curve EASY

Differentiate $y = 4x^3 - x$: $\dfrac{dy}{dx} = 12x^2 - 1$ **M1**

Evaluate at $x = 1$: $\left.\dfrac{dy}{dx}\right|_{x=1} = 12(1)^2 - 1 = 11$, and $y = 4(1)^3 - 1 = 3$ **A1**

Use $y - y_1 = m(x - x_1)$ with $m = 11$ and point $(1, 3)$: $y - 3 = 11(x - 1) \Rightarrow y = 11x - 8$ **A1**

Note: Substitute $x = 1$ into the original function to find the y-coordinate; do not assume the point lies on the x-axis.

Q7 — Decreasing Interval for Water Depth EASY

(a) Differentiate $D(t) = t^2 - 8t + 20$ term by term: $D'(t) = 2t - 8$ **A1**

(b) The depth is decreasing when $D'(t) < 0$: $2t - 8 < 0 \Rightarrow t < 4$ **M1**

Combined with the domain $0 \leq t \leq 8$, the depth is decreasing for $0 \leq t < 4$ hours. **A1**

Note: The domain restriction $0 \leq t \leq 8$ must be applied; writing $t < 4$ without the lower bound is incomplete.

Q8 — Normal to a Quadratic Curve EASY

Differentiate $y = 2x^2 - 8x + 5$: $\dfrac{dy}{dx} = 4x - 8$, so $\left.\dfrac{dy}{dx}\right|_{x=3} = 4(3) - 8 = 4$ **M1**

Check the point of contact by substituting $x = 3$ into the curve: $y(3) = 2(3)^2 - 8(3) + 5 = -1$, so $P = (3, -1)$.

The gradient of the normal is the negative reciprocal of the tangent gradient: $m_{\text{normal}} = -\dfrac{1}{4}$ **A1**

Use $y - y_1 = m(x - x_1)$ with $m = -\dfrac{1}{4}$ and point $(3, -1)$: $y - (-1) = -\dfrac{1}{4}(x - 3) \Rightarrow y = -1/4x - 1/4$ **A1**

Note: The normal uses the negative reciprocal of the tangent gradient, not the tangent gradient itself. Also always check that the given point actually lies on the curve before using it — substituting $x = 3$ gives $y = -1$, so the point of contact is $(3, -1)$.

Q9 — Increasing Interval of a Cubic EASY

(a) Differentiate $f(x) = x^3 - 6x^2 + 9x + 1$ term by term: $f'(x) = 3x^2 - 12x + 9$ **A1**

(b) f is increasing where $f'(x) > 0$. Set $f'(x) = 0$: GDC equation solver gives $x = 1$ or $x = 3$. **M1**
Test the sign of $f'(x)$: for $x < 1$, $f'(0) = 9 > 0$; for $1 < x < 3$, $f'(2) = -3 < 0$; for $x > 3$, $f'(4) = 9 > 0$.
The function is increasing for $x < 1$ and $x > 3$. **A1**

Note: The function is increasing on two separate intervals; writing only one of them would lose the mark.

Q10 — Particle at Rest EASY

The velocity of the particle is the derivative of displacement with respect to time: $s'(t) = 6t - 6$ **M1**

The particle is momentarily at rest when the velocity equals zero: $6t - 6 = 0 \Rightarrow t = 1$ second **A1A1**

Note: The question asks for the time, not the displacement; substituting $t = 1$ back into $s(t)$ to give a distance answer would not earn the final mark.

Q11 — Rate of Change of Depth Model MEDIUM

(a) Differentiate each term of $d(t)$ using the power rule: $d'(t) = 3(0.5)t^2 - 2(6)t + 18 = 1.5t^2 - 12t + 18$ **M1A1**

(b) Substitute $t = 3$: $d'(3) = 1.5(9) - 12(3) + 18 = 13.5 - 36 + 18 = -4.5$ m h^{-1} **M1A1**

Note: A sign error here (writing $+12t$ instead of $-12t$) costs the A1; always check the sign when differentiating a negative coefficient.

Q12 — Tangent to a Quadratic at a Point MEDIUM

(a) Differentiate $f(x) = 3x^2 - 8x + 5$: $f'(x) = 6x - 8$. Substitute $x = 4$: $f'(4) = 6(4) - 8 = 16$ **M1A1**

(b) Find the y-coordinate of the point of tangency: $f(4) = 3(4)^2 - 8(4) + 5 = 48 - 32 + 5 = 21$, so the point is $(4, 21)$.
Use point–slope form $y - 21 = 16(x - 4)$: $y = 16x - 64 + 21 \Rightarrow y = 16x - 43$ **M1A1**

Note: Always recompute $f(a)$ freshly before forming the tangent equation — do not read the y-coordinate off an unchecked diagram; here $f(4) = 21$, not the value shown on an uncorrected sketch.

Q13 — Stationary Points of a Cubic MEDIUM

(a) Differentiate and set equal to zero: $f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3) = 0$.
GDC: Equation solver or graph of $f'(x)$ to confirm roots. **M1**
$x = 1$ and $x = 3$ **A1**

(b) Use a sign table of $f'(x)$ to classify each point.
At $x = 0$: $f'(0) = 9 > 0$; at $x = 2$: $f'(2) = 12-24+9 = -3 < 0$; at $x = 4$: $f'(4) = 48-48+9 = 9 > 0$. **M1**
Since f' changes from $+$ to $-$ at $x = 1$, there is a local maximum at $x = 1$. Since f' changes from $-$ to $+$ at $x = 3$, there is a local minimum at $x = 3$. **A1**
Note: Asserting "local maximum" or "local minimum" without showing the sign change of f' earns no marks; the classification evidence must be explicit.

Q14 — Maximum Profit Quadratic Model MEDIUM

(a) Differentiate $P(x) = -2x^2 + 12x - 7$ and set $P'(x) = 0$: $-4x + 12 = 0 \Rightarrow x = 3$ **M1A1**
GDC: Graph $P(x)$ on $[0, 6]$ and use maximum feature to confirm $x = 3$.

(b) Substitute $x = 3$ into $P(x)$: $P(3) = -2(9) + 12(3) - 7 = -18 + 36 - 7 = 11$ **M1A1**
The maximum profit is \$11 000.
Note: The units of P are thousand dollars; writing \$11 instead of \$11 000 loses the contextual accuracy of the answer.

Q15 — Differentiating a Negative Power MEDIUM

(a) Rewrite $g(x)$ using a negative index before differentiating: $g(x) = 4x^{-2} + 3x$.
Apply the power rule term by term: $g'(x) = -8x^{-3} + 3 = -8/x^3 + 3$ **M1A1**
Note: Differentiating $4x^{-2}$ as $+8x^{-3}$ (sign error on a negative power) is the most common trap here; the rule gives $n \cdot ax^{n-1} = (-2)(4)x^{-3} = -8x^{-3}$.

(b) Substitute $x = 2$: $g'(2) = -8/8 + 3 = -1 + 3 = 2$ **M1A1**

Q16 — Decreasing Interval and Local Minimum MEDIUM

(a) Differentiate $h(x) = x^3 - 3x^2 - 9x + 4$ and solve $h'(x) < 0$: $h'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$ **M1**
GDC: Graph $h'(x)$ and identify the interval where it is negative.
$h'(x) < 0$ for $-1 < x < 3$, so h is decreasing on $(-1, 3)$. **A1**

(b) The local minimum occurs at $x = 3$ (where h' changes from $-$ to $+$): $h(3) = 27 - 27 - 27 + 4 = -23$ **M1A1**
The local minimum is at $(3, -23)$.
Note: Giving the local maximum coordinates $(-1, 9)$ instead of the local minimum when the question specifically asks for the minimum is a common mix-up; always check whether f' changes $- \to +$ (minimum) or $+ \to -$ (maximum).

Q17 — Normal to a Quadratic MEDIUM

(a) First find the tangent gradient: $f'(x) = 4x - 5$, so $f'(3) = 12 - 5 = 7$ **A1**

The gradient of the normal is the negative reciprocal: $m_{\text{normal}} = -\dfrac{1}{7}$ **M1**

Find the point of contact: $f(3) = 2(9) - 15 + 1 = 4$, so the point is $(3, 4)$.

Use point–slope form: $y - 4 = -\dfrac{1}{7}(x - 3) \Rightarrow y = -1/7x + 31/7$ **M1A1**

(b) The gradient of the tangent to C at $x = 3$ is $f'(3) = 7$ (found above). **A1**

Note: Using the tangent gradient 7 (instead of its negative reciprocal $-\frac{1}{7}$) for the normal equation is the key trap in normal-line problems; always invert and negate.

Q18 — Rate of Change of Projectile Height MEDIUM

(a) Differentiate $h(t) = -4.9t^2 + 14.7t + 1.2$ term by term: $h'(t) = -9.8t + 14.7$ **A1**

(b) At maximum height, $h'(t) = 0$: $-9.8t + 14.7 = 0 \Rightarrow t = 14.7/9.8 = 1.5$ s **M1A1**

GDC: Graph $h(t)$ and use maximum to confirm $t = 1.5$.

(c) $h'(t) < 0$ means the height of the ball is decreasing; the ball is falling after reaching its maximum height. **R1**

Note: Show the unrounded division $14.7 \div 9.8 = 1.5$ explicitly rather than rounding prematurely.

Q19 — Finding Points with a Given Gradient MEDIUM

(a) Differentiate $f(x) = x^3 - 12x + 2$ and set $f'(x) = 0$: $3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$ **A1**

(b) Set $f'(x) = 24$: $3x^2 - 12 = 24 \Rightarrow 3x^2 = 36 \Rightarrow x^2 = 12$ **M1**

$x = \pm\sqrt{12} = \pm 2\sqrt{3} \approx \pm 3.46$ **A1A1**

GDC: Graph $y = f'(x) = 3x^2 - 12$ and $y = 24$; use intersection to confirm $x \approx \pm 3.464$.

Note: Forgetting the negative solution $x = -2\sqrt{3}$ (since $x^2 = 12$ has two roots) costs one A1; always consider both square roots when solving $x^2 = k$.

Q20 — Optimisation with AG Firewall MEDIUM

(a) Let the length be l metres. The perimeter condition gives $2x + 2l = 40$, so $l = 20 - x$.

The area is: $A = x \cdot l = x(20 - x) = 20x - x^2$ **AG**

No marks awarded for the printed result; marks are for the method leading to it. **(M1)**

(b) Differentiate $A(x) = 20x - x^2$ and set $A'(x) = 0$: $20 - 2x = 0 \Rightarrow x = 10$ **M1A1**

GDC: Graph $A(x)$ on $[0, 20]$ and use maximum to confirm $x = 10$.

Substitute: $A(10) = 20(10) - 100 = 200 - 100 = 100$ m^2 **A1**

Note: Using the AG expression from part (a) is compulsory here (the "hence" chain); deriving a different expression from scratch and getting a different answer loses all marks in part (b).

Q21 — Polynomial Profit Optimisation HARD

(a) Differentiating term by term using the power rule: $P'(x) = -6x^2 + 60x - 100$ **A1**

(b) Set $P'(x) = 0$: GDC equation solver (Algebra → Solve) gives $x \approx 2.11$ and $x \approx 7.89$. **M1**
To classify, evaluate $P'(x)$ either side of each stationary point:
At $x \approx 2.11$: $P'(2) = -24 + 120 - 100 = -4 < 0$; $P'(3) = -54 + 180 - 100 = 26 > 0$, so local minimum at $x \approx 2.11$.
At $x \approx 7.89$: $P'(7) = -294 + 420 - 100 = 26 > 0$; $P'(9) = -486 + 540 - 100 = -46 < 0$, so local maximum at $x \approx 7.89$.
Within $2 \leq x \leq 20$, also check endpoints: $P(2) = -16 + 120 - 200 + 80 = -16$; $P(20) = -16000 + 12000 - 2000 + 80 = -5920$.
The local maximum at $x \approx 7.89$ exceeds both endpoint values, so it is the global maximum on the domain. **A1**
R1: Since P' changes from positive to negative at $x \approx 7.89$, this is a local maximum, confirmed as the global maximum by the endpoint check. **R1**

Note: A common trap is to ignore the endpoints of the restricted domain $[2, 20]$; always evaluate P at both endpoints and compare with the local maximum value.

(c) Substituting the unrounded stationary value $x = 5 + \dfrac{5\sqrt{3}}{3} \approx 7.887$ into $P(x)$ gives $P_{\text{max}} = 176.225$ (thousand dollars).
Maximum profit ≈ \$176 225, which to 3 significant figures is \$176,000. **A1**

Note: The function gives P in thousands of dollars; forgetting to multiply by 1000 loses the context mark. Keep the unrounded critical value until the final substitution — rounding x to 7.89 early still gives the same result to 3 s.f.

Q22 — Differentiating Negative Powers; Increasing Function HARD

(a) Rewrite $f(x)$ using negative powers before differentiating: $f(x) = 3x^2 - 48x^{-2}$.
Differentiating term by term using the power rule ($\frac{d}{dx}[x^n] = nx^{n-1}$):
$f'(x) = 6x - 48 \cdot (-2)x^{-3} = 6x + 96x^{-3} = 6x + 96/x^3$ **M1**
This matches the printed target. **AG**

Note: The most frequent error is computing $\frac{d}{dx}[-48x^{-2}] = +96x^{-3}$ correctly but then writing $-96x^{-3}$ (sign error on the negative power); show the multiplication step explicitly.

(b) For all $x > 0$: both terms of $f'(x) = 6x + 96/x^3$ are strictly positive since $x > 0$, so $f'(x) > 0$ for all $x > 0$. **M1**
R1: Since $f'(x) > 0$ for all x in the domain, f is strictly increasing on $x > 0$. **R1**

(c) First find $f(2)$ and $f'(2)$: $f(2) = 3(4) - 48/4 = 12 - 12 = 0$, $f'(2) = 6(2) + 96/8 = 12 + 12 = 24$.
Tangent equation using $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (2, 0)$ and $m = 24$: $y = 24(x - 2) \Rightarrow y = 24x - 48$ **A1**

Q23 — Optimisation with AG Firewall; Domain Validity HARD

(a) The side parallel to the wall has length $60 - 2x$ metres. The area is: $A = x(60 - 2x) = 60x - 2x^2$ **AG**
This is the printed target; no marks awarded for the statement alone.

(b) Differentiating $A = 60x - 2x^2$ with respect to x: $\dfrac{dA}{dx} = 60 - 4x$ **A1**

(c) Set $\dfrac{dA}{dx} = 0$: $60 - 4x = 0 \Rightarrow x = 15$ **A1**

(d) Hence (using $x = 15$ from part (c)): $A = 60(15) - 2(15)^2 = 900 - 450 = 450$ m^2 **A1**

Note: The "hence" instruction makes it compulsory to use $x = 15$ from part (c); restarting with a different method earns no credit.

(e) When $x = 35$, the third side has length $60 - 2(35) = -10$ metres, which is negative; a length cannot be negative, so the model is not appropriate for $x = 35$. **A1**

Note: Simply stating "it is outside the domain" is insufficient; state explicitly that the resulting side length becomes negative, which is physically impossible.

Q24 — Stationary Points and Normal and Increasing Intervals HARD

(a) $g'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0$ gives $x = 1$ and $x = 3$, consistent with the labelled diagram.
Local maximum: $(1, 6)$; Local minimum: $(3, 2)$ **A1**

(b) First find $g'(x)$ and evaluate at $x = 2$: $g'(2) = 3(4) - 12(2) + 9 = -3$ **M1**
Gradient of normal: $m_n = -1/(-3) = 1/3$ **M1**
Point on curve: $g(2) = 8 - 24 + 18 + 2 = 4$, so the point is $(2, 4)$.
Normal equation: $y - 4 = \dfrac{1}{3}(x - 2) \Rightarrow y = 1/3x + 10/3$ **A1**

Note: Using the tangent gradient (-3) instead of its negative reciprocal for the normal is the most common error; always show the negative reciprocal step explicitly.

(c) g is increasing where $g'(x) > 0$, i.e. $3(x - 1)(x - 3) > 0$, which holds for $x < 1$ or $x > 3$. **A1**

Q25 — Negative Power Rewrite; Minimum of a Rational Model HARD

(a) Rewrite $T(t)$ expressing $120/t$ as a negative power: $T(t) = 120t^{-1} + 4t - 10$.
Differentiate term by term: $T'(t) = -120t^{-2} + 4 = -120/t^2 + 4$ **M1A1**

Note: Writing $\frac{d}{dt}[120t^{-1}] = +120t^{-2}$ (forgetting the negative sign from the power rule) is the flagged sign-error trap; show the step $(-1) \times 120 = -120$ explicitly.

(b) Set $T'(t) = 0$: $-120/t^2 + 4 = 0 \Rightarrow t^2 = 30 \Rightarrow t = \sqrt{30} \approx 5.48$ min **M1**
Sign of $T'(t)$: for $0 < t < \sqrt{30}$, $T'(t) < 0$ (decreasing); for $t > \sqrt{30}$, $T'(t) > 0$ (increasing).
R1: Since T' changes from negative to positive at $t = \sqrt{30}$, this is a local minimum. **R1**

(c) Minimum temperature (substituting $t = \sqrt{30}$, unrounded): $T(\sqrt{30}) = 120/\sqrt{30} + 4\sqrt{30} - 10 =$

$4\sqrt{30} + 4\sqrt{30} - 10 = 8\sqrt{30} - 10$.
$T_{\text{min}} = 8\sqrt{30} - 10 \approx 43.8 - 10 = 33.8\,^{\circ}\text{C}$ **A1**

Q26 — Gradient and Tangent Equation and Stationary Points HARD

(a) Differentiate $h(x) = 2x^3 - 9x^2 + 12x - 1$ term by term: $h'(x) = 6x^2 - 18x + 12$ **A1**

(b) Evaluate at $x = 3$: $h'(3) = 6(9) - 18(3) + 12 = 54 - 54 + 12 = 12$ **A1**

(c) Find the y-coordinate: $h(3) = 2(27) - 9(9) + 12(3) - 1 = 54 - 81 + 36 - 1 = 8$, giving point $(3, 8)$.
Tangent equation with $m = 12$: $y - 8 = 12(x - 3) \Rightarrow y = 12x - 28$ **M1A1**

(d) Set $h'(x) = 0$: $6x^2 - 18x + 12 = 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0$: $x = 1$ and $x = 2$ **A1**

Note: A common error is to use the y-value from the diagram as the gradient; always compute $h'(3)$ algebraically or via GDC.

Q27 — Cubic Height Model; Classification by Sign Diagram HARD

(a) Differentiate $H(t) = -t^3 + 9t^2 - 15t + 30$ term by term: $H'(t) = -3t^2 + 18t - 15$ **A1**

(b) Set $H'(t) = 0$ and factorise: $-3t^2 + 18t - 15 = 0 \Rightarrow t^2 - 6t + 5 = 0 \Rightarrow (t - 1)(t - 5) = 0$: $t = 1$ s and $t = 5$ s **A1**

(c) Sign diagram for $H'(t) = -3(t - 1)(t - 5)$:

Interval	$t < 1$	$t = 1$	$1 < t < 5$	$t = 5$	$t > 5$
$H'(t)$	$-$	0	$+$	0	$-$
H	decreasing		increasing		decreasing

M1

R1: At $t = 1$, H' changes from negative to positive $\Rightarrow$ local minimum; at $t = 5$, H' changes from positive to negative $\Rightarrow$ local maximum. **R1**

Note: Asserting "local maximum" or "local minimum" without evidence from the sign of H' on either side earns no marks; the sign diagram is compulsory.

(d) Compare H at the local max and both endpoints of $[0, 8]$: $H(5) = -125 + 225 - 75 + 30 = 55$; $H(0) = 30$; $H(8) = -512 + 576 - 120 + 30 = -26$.
Maximum height is 55 metres at $t = 5$ s. **A1**

Q28 — Parameter Recovery from Derivative and Function Conditions HARD

(a) The printed working "$a + b = 0$ and $a + b = 4$" contains an error: differentiating ax^2 gives $2ax$, not ax, so the first equation is missing a factor of 2. Find $f'(x)$ correctly first: $f'(x) = 2ax + b$.
Condition $f'(1) = 0$: $2a(1) + b = 0$, so $2a + b = 0$ **M1**

Condition $f(1) = 7$: $a(1)^2 + b(1) + 3 = 7$, so $a + b = 4$
The two correct equations are $2a + b = 0$ and $a + b = 4$. **A1**

Note: Always differentiate before substituting; omitting the coefficient 2 from differentiating ax^2 is the flagged error in this question.

(b) Subtract the second equation from the first to eliminate b: $(2a + b) - (a + b) = 0 - 4 \Rightarrow a = -4$ **M1**
Substitute into $a + b = 4$: $-4 + b = 4 \Rightarrow b = 8$ **A1**

(c) The stationary point occurs where $f'(x) = 0$, i.e. at $x = 1$ (given). The y-coordinate is $f(1) = 7$ (given). Stationary point: $(1, 7)$ **A1**

Q29 — Cubic Stationary Points and Normal at a Given Point HARD

(a) Differentiate $y = x^3 - 3x^2 - 9x + 5$ term by term: $\frac{dy}{dx} = 3x^2 - 6x - 9$ **A1**

(b) Set $\frac{dy}{dx} = 0$: $3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0$ **M1**
$x = 3$: $y = 27 - 27 - 27 + 5 = -22$. $x = -1$: $y = -1 - 3 + 9 + 5 = 10$.
Stationary points: $(-1, 10)$ and $(3, -22)$ **A1**

(c) Gradient of curve at $P(-1, 10)$: $\left.\frac{dy}{dx}\right|_{x=-1} = 3(1) - 6(-1) - 9 = 0$ **M1**
Since the tangent gradient is 0, the tangent is horizontal, so the normal is vertical with equation $x = -1$. **A1**

Note: When the tangent gradient is 0, the normal gradient is undefined (vertical line); the normal equation is $x = -1$, not a sloped line.

Q30 — Cylinder Volume Optimisation with AG Firewall HARD

(a) The total surface area of a closed cylinder is $2\pi r^2 + 2\pi rh = 300\pi$. Divide through by π: $2r^2 + 2rh = 300 \Rightarrow h = \frac{300 - 2r^2}{2r} = \frac{150}{r} - r$ **M1**
Volume of cylinder: $V = \pi r^2 h$. Substitute the expression for h: $V = \pi r^2 \left(\frac{150}{r} - r\right) = 150\pi r - \pi r^3$ **AG**

Note: The surface area formula must include both circular ends ($2\pi r^2$) since the cylinder is closed; omitting one end gives an incorrect expression for h and the AG cannot be reached.

(b) Differentiate $V = 150\pi r - \pi r^3$ with respect to r: $\frac{dV}{dr} = 150\pi - 3\pi r^2$ **A1**

(c) Set $\frac{dV}{dr} = 0$: $150\pi - 3\pi r^2 = 0 \Rightarrow r^2 = 50 \Rightarrow r = \sqrt{50} = 5\sqrt{2} \approx 7.07$ cm **M1**
Sign of $\frac{dV}{dr}$: for $r < 5\sqrt{2}$, $\frac{dV}{dr} > 0$; for $r > 5\sqrt{2}$, $\frac{dV}{dr} < 0$.
R1: Since $\frac{dV}{dr}$ changes from positive to negative at $r = 5\sqrt{2}$, this is a local maximum, so the volume is maximised at $r = 5\sqrt{2} \approx 7.07$ cm. **R1**

Note: Only the positive root $r = 5\sqrt{2}$ is physically valid since $r > 0$; discarding the negative root must be stated explicitly.