IB Math AI SL — Topic 5

Integration

Trapezoidal Rule · Antiderivatives & $+c$ · Areas Using a GDC

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:____________________________ Date:______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Power Rule (Indefinite):** $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- **Constant Multiple & Sum:** $\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$
- **Constant of Integration:** given $f'(x)$ and a point (x_0, y_0) on the curve, substitute to find c in $f(x) = \int f'(x)\,dx + c$
- **Definite Integral (Area):** $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$
- **Trapezoidal Rule (n strips, $n+1$ values):** $\int_a^b f(x)\,dx \approx \dfrac{h}{2}[y_0+2(y_1+y_2+\cdots+y_{n-1})+y_n], \quad h = \dfrac{b-a}{n}$
- **Negative Powers (antiderivative):** $\int x^{-n}\,dx = \dfrac{x^{-n+1}}{-n+1} + c, \quad n \neq 1$; e.g. $\int x^{-2}\,dx = -x^{-1} + c$
- **Over/Under Estimate (concavity):** trapezoidal rule *overestimates* when the curve is concave up ($f''(x) > 0$); *underestimates* when concave down ($f''(x) < 0$)

GDC Tips

- **Definite integral / area:** `Menu` $\to$ `Calculus` $\to$ `Numerical Integral`; enter $f(x)$, lower limit a, upper limit b. The calculator returns $\int_a^b f(x)\,dx$ directly.
- **Storing an unrounded integral for later use:** press `Store` $\to$ `var` (e.g. `A`) immediately after the numerical-integral result appears; use `A` in subsequent calculations to avoid premature rounding.
- **Checking an antiderivative:** `Menu` $\to$ `Calculus` $\to$ `Integral` with no limits gives the symbolic antiderivative (omits $+c$); confirm your hand answer matches.
- **Graphing to identify the region:** plot $f(x)$ in `Graph` page, then use `Menu` $\to$ `Analyze Graph` $\to$ `Integral` and click the lower and upper bounds to shade and compute the area visually.
- **Table of values for the trapezoidal rule:** define $f(x)$ in a `Lists & Spreadsheet` column with formula `=f(x_val)`, fill the x-values in column A and read off the y-values in column B; this eliminates arithmetic slips in the table.
- **Finding the constant of integration:** after obtaining the general antiderivative symbolically, use `Calculator` page to substitute the given point and solve for c via `Menu` $\to$ `Algebra` $\to$ `Solve`.

Common Mistakes to Avoid

1. **Forgetting $+c$ on every indefinite integral.** The constant of integration must appear on every antiderivative that has no limits. Its omission loses the accuracy mark outright, even if all other working is correct.
2. **Strips versus values miscount in the trapezoidal rule.** n strips require $n+1$ function values ($y_0, y_1, \dots, y_n$). Writing only n values means the final ordinate is missing and the bracket structure collapses, costing the method mark.
3. **Incorrect bracket structure in the trapezoidal rule.** Only y_0 and y_n appear once; every interior value $y_1, \dots, y_{n-1}$ is multiplied by 2. Doubling the endpoints (or not doubling the middle values) gives a wrong numerical answer.
4. **Premature rounding carried through chained parts.** If an intermediate result (e.g. the value of c, or an area) is rounded to 3 s.f. before use, the final answer drifts and loses accuracy marks. Always store the unrounded value and round only the stated final answer.
5. **Wrong sign when integrating a negative power.** For example, $\displaystyle\int x^{-2}\,dx = -x^{-1}+c$, *not* $+x^{-1}+c$. The power rule gives $\frac{x^{-1}}{-1}$; the negative denominator flips the sign and is the most common error on negative-power terms.
6. **Incorrect over/under-estimate judgement.** The concavity argument must be stated: if $f''(x) > 0$ on $[a,b]$ the curve bends *upward*, so the trapezoids lie *above* the curve and the rule *overestimates*. Reversing this (or giving no reason) loses the reasoning mark R1.

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Section A — Easy

EASY

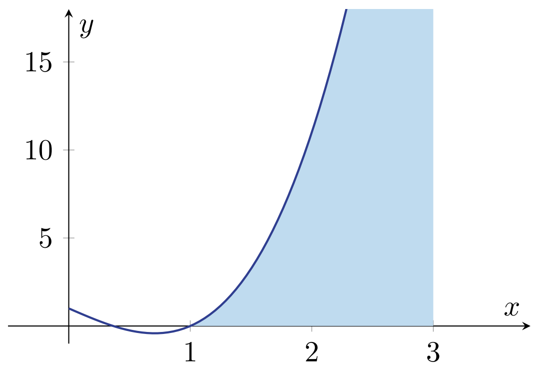
1. Find $\int (3x^2 + 4x - 5)\,dx$. **[3 marks]**

2. Find $\int \left(6x^2 - \frac{2}{x^3}\right) dx$. **[3 marks]**

3. The gradient of a curve is given by $f'(x) = 4x - 3$. The curve passes through the point $(2, 5)$. Find $f(x)$. **[3 marks]**

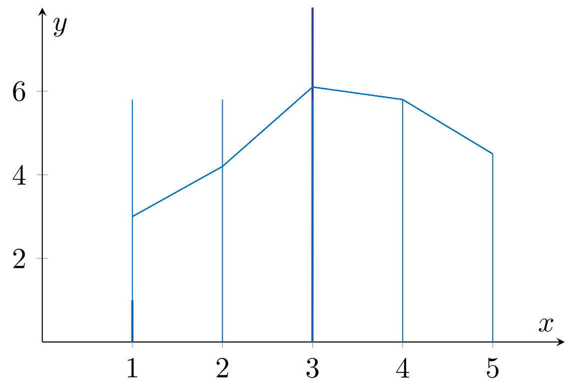
4. Find $\int (5x^4 + 2x^{-2})\, dx$. **[3 marks]**

5. A function is given by $f(x) = 2x^3 - 3x + 1$. The shaded region below is bounded by the curve, the x-axis, and the lines $x = 1$ and $x = 3$. Use your GDC to find the area of this shaded region. **[3 marks]**



6. A table of values for a function $g(x)$ is shown below. Use the trapezoidal rule with four strips to estimate $\int_1^5 g(x)\,dx$. **[3 marks]**

x	1	2	3	4	5
$g(x)$	3.0	4.2	6.1	5.8	4.5

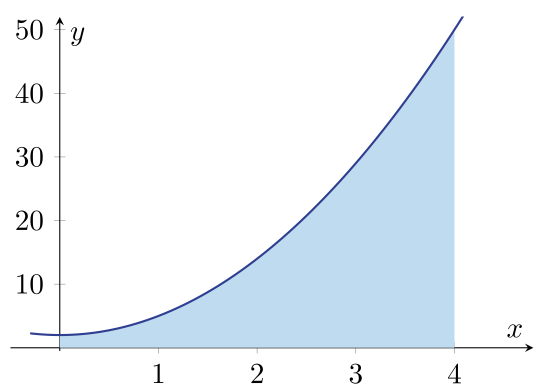


7. Find $\int_0^3 (x^2+2)\,dx$ using your GDC. **[3 marks]**

8. The gradient of a curve is given by $f'(x) = 6x^2 - 4x + 1$. The curve passes through the point $(1, 3)$. Find the value of $f(0)$. **[3 marks]**

9. Find $\displaystyle\int (8x^3 - 3x^2 + 7)\,dx$. **[3 marks]**

10. The function $h(x) = 3x^2 + 2$ is defined for $0 \le x \le 4$. The shaded region is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 4$. Use your GDC to find the exact area of this shaded region. **[3 marks]**



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Section B — Medium

MEDIUM

11. A biologist models the growth rate of a bacterial colony. The rate of change of the colony size N (thousands of bacteria) with respect to time t (hours) is given by

$$\frac{dN}{dt} = 3t^2 - 4t + 5.$$

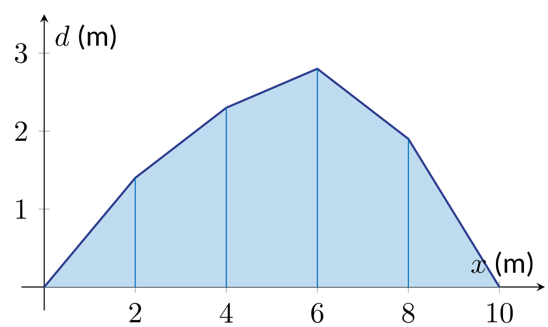
At $t = 1$ hour, the colony size is $N = 6$ thousand bacteria.

(a) Find an expression for $N(t)$. **[3 marks]**

(b) Find the colony size when $t = 3$ hours. **[1 marks]**

12. A river has a cross-section measured at six equally-spaced points, 2 m apart. The depths d (metres) at each position x (metres) are given in the table below. Use the trapezoidal rule with all five strips to estimate the cross-sectional area of the river. **[4 marks]**

x (m)	0	2	4	6	8	10
d (m)	0	1.4	2.3	2.8	1.9	0



13. A curve has gradient function $f'(x) = 6x^2 - \dfrac{4}{x^3}$, where $x > 0$. The curve passes through the point $(1, 9)$.

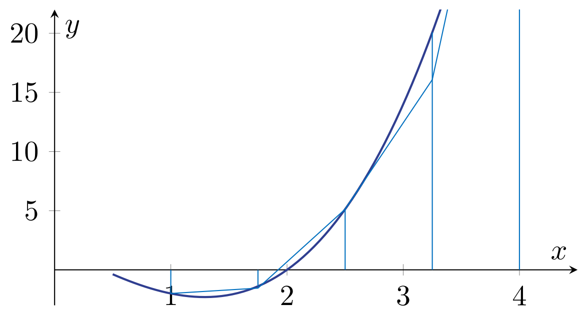
(a) Find $f(x)$. **[3 marks]**

(b) Find the value of $f(2)$. **[1 marks]**

14. A function is defined as $g(x) = x^3 - 5x + 2$ for $1 \leq x \leq 4$.

(a) Use the trapezoidal rule with four strips of equal width to estimate $\int_1^4 g(x)\,dx$, showing the table of values used. **[3 marks]**

(b) State whether your estimate is an overestimate or underestimate of the true value. Justify your answer. **[1 marks]**



15. The velocity of a remote-controlled car is modelled by $v(t) = 4t - t^2$ metres per second, for $0 \leq t \leq 4$ seconds, where t is time in seconds.

(a) Write down the definite integral that gives the total distance travelled by the car from $t = 0$ to $t = 4$. **[1 marks]**

(b) Calculate this distance using a GDC. **[2 marks]**

(c) State one reason why the model may not be appropriate for $t > 4$. **[1 marks]**

16. A curve satisfies $\dfrac{dy}{dx} = 5x^4 - 3x^2 + 2$. The curve passes through the point $(2,\ 30)$.

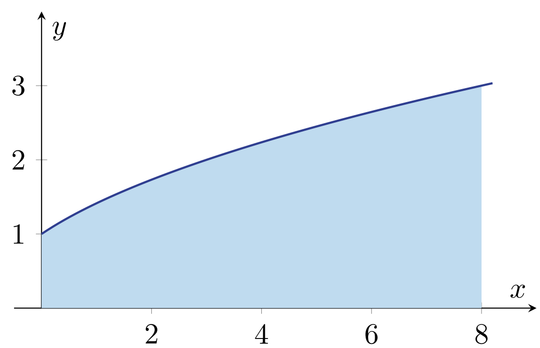
(a) Find y in terms of x. **[3 marks]**

(b) Write down the y-intercept of the curve. **[1 marks]**

17. The function $h(x) = \sqrt{x+1}$ is defined for $0 \leq x \leq 8$.

(a) Calculate the area of the region enclosed by the curve $y = h(x)$, the x-axis, and the lines $x = 0$ and $x = 8$, using a GDC. **[2 marks]**

(b) Use the trapezoidal rule with four strips to estimate this area. Show your working clearly. **[2 marks]**



18. The marginal cost function for producing q units of a product is $C'(q) = 0.6q^2 - 8q + 40$ dollars per unit, where $q \geq 0$. The fixed cost (cost of producing zero units) is $\$200$.

(a) Find the total cost function $C(q)$. **[3 marks]**

(b) Calculate the total cost of producing 10 units. **[1 marks]**

19. A particle moves along a straight line. Its acceleration is $a(t) = 6t - 4$ metres per second squared, where t is time in seconds, $t \geq 0$. The particle's velocity at $t = 0$ is $v = 7$ m s^{-1}.

(a) Find an expression for $v(t)$, the velocity of the particle at time t. **[3 marks]**

(b) Find the velocity when $t = 5$ seconds. **[1 marks]**

20. The area enclosed by the curve $y = x^2 - 4x + 3$, the x-axis, and the lines $x = 0$ and $x = 3$ is to be found using a GDC.

(a) Sketch the curve for $0 \leq x \leq 3$, labelling any x-intercepts. **[1 marks]**

(b) Calculate, using a GDC, the total area of the region between the curve and the x-axis for $0 \leq x \leq 3$. **[3 marks]**

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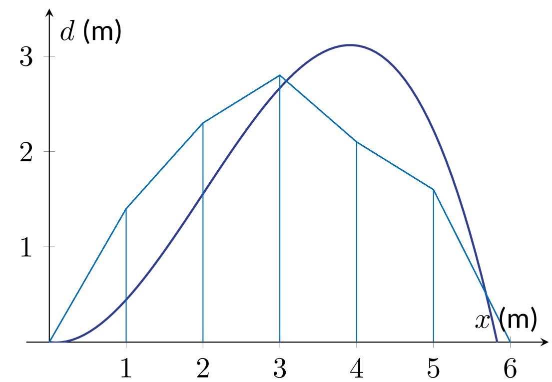
Section C — Hard

HARD

21. A botanist models the cross-sectional area of a river using measurements taken at 1-metre intervals across its 6-metre width. The depths d (in metres) at each measurement point are given in the table below.

x (m)	0	1	2	3	4	5	6
d (m)	0	1.4	2.3	2.8	k	1.6	0



The trapezoidal rule with 6 strips gives a cross-sectional area estimate of 10.1 m^2.

(a) Show that $k = 2.1$. **[2 marks]**

(b) The actual cross-sectional area, found using a GDC, is 10.44 m^2 (3 s.f.). Determine whether the trapezoidal estimate is an overestimate or an underestimate. Justify your answer. **[2 marks]**

(c) Calculate the percentage error of the trapezoidal estimate. **[1 marks]**

22. A particle moves along a straight line. Its velocity v (in m s^{-1}) at time t seconds is given by

$$v(t) = 3t^2 - 8t + 4, \quad 0 \leq t \leq 4.$$

(a) Find an expression for the displacement $s(t)$ of the particle from its starting position, given that $s(0) = 5$. **[3 marks]**

(b) Find the displacement of the particle at $t = 4$. **[1 marks]**

(c) Write down the value of t at which the particle returns to its starting position (i.e. $s(t) = 5$). Justify that there is exactly one such value in $0 < t \leq 4$. **[1 marks]**

23. The function f is defined by $f'(x) = 6x^{-2} - 4x + 3$ for $x > 0$. The graph of f passes through the point $(1, 9)$.

(a) Find $f(x)$. **[3 marks]**

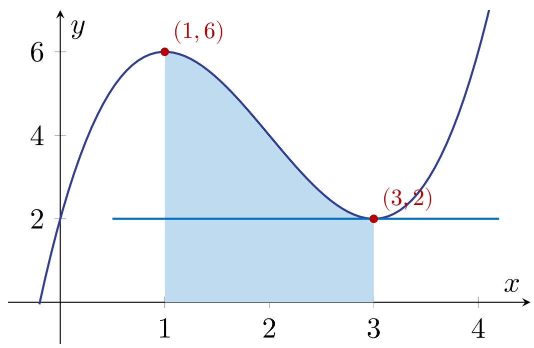
(b) Find $f(3)$. **[1 marks]**

(c) State why the expression for $f(x)$ is not valid for $x \leq 0$. **[1 marks]**

24. Let $g(x) = x^3 - 6x^2 + 9x + 2$.

(a) Show that $g'(x) = 3(x-1)(x-3)$. **[2 marks]**

(b) Find the area of the region enclosed by the curve $y = g(x)$ and the horizontal line $y = 2$ for $1 \le x \le 3$. **[3 marks]**



25. A photographer models the light intensity I (in lux) across a studio wall at height x metres above the floor by

$$I(x) = 80 + 12x - 3x^2, \quad 0 \leq x \leq 4.$$

(a) Use the trapezoidal rule with 4 equal strips to estimate $\displaystyle\int_0^4 I(x)\,dx$. **[3 marks]**

(b) Use a GDC to find the exact value of $\displaystyle\int_0^4 I(x)\,dx$. **[1 marks]**

(c) Determine, with justification, whether the trapezoidal rule gives an overestimate or an underestimate for this function on $[0, 4]$. **[1 marks]**

26. The rate of change of temperature T (°C) inside a greenhouse t hours after midnight is modelled by

$$\frac{dT}{dt} = 0.4t - 0.05t^2, \quad 0 \leq t \leq 12.$$

At $t = 2$, the temperature inside the greenhouse is 18 °C.

(a) Find an expression for $T(t)$. **[3 marks]**

(b) Find the temperature at $t = 12$. **[1 marks]**

(c) Find the value of t in $0 \leq t \leq 12$ at which the temperature is greatest. Justify your answer. **[1 marks]**

27. The function h satisfies $h'(x) = 5x^{-3} + 2x$ for $x > 0$. It is known that $\displaystyle\int_1^a h'(x)\,dx = 20$ where $a > 1$.

(a) Show that $\displaystyle\int_1^a h'(x)\,dx = a^2 - \frac{5}{2a^2} + \frac{3}{2}$. **[3 marks]**

(b) Hence find the value of a. **[2 marks]**

28. A landscape architect records the height y (in metres) of a sculpted grass mound at horizontal distances x metres from its base. The values are given in the table.

x (m)	0	1.5	3	4.5	6	7.5
y (m)	0	1.2	2.1	2.4	1.8	0

(a) Use the trapezoidal rule with 5 strips to estimate the cross-sectional area of the mound. **[3 marks]**

(b) A GDC gives the true area as 9.23 m^2 (3 s.f.). Calculate the percentage error of the trapezoidal estimate, giving your answer to 3 significant figures. **[2 marks]**

29. Let $f(x) = px^2 + qx$ where p and q are constants. It is given that $\int_0^2 f(x)\,dx = 10$ and that $f'(1) = 7$.

(a) Show that $\dfrac{8p}{3} + 2q = 10$. **[2 marks]**

(b) Find the values of p and q. **[2 marks]**

(c) Hence find the area enclosed between $y = f(x)$ and the x-axis for $0 \leq x \leq 2$. **[1 marks]**

30. A marine biologist models the population density P (organisms per litre) of a plankton bloom at depth x metres below the surface by

$$P(x) = 40 + 18x^{1/2} - 3x, \quad 0 \leq x \leq 16.$$

(a) Find $\displaystyle\int P(x)\,dx$. **[2 marks]**

(b) Use a GDC to find $\displaystyle\int_0^{16} P(x)\,dx$, giving your answer to 4 significant figures. **[1 marks]**

(c) Find the depth c, where $0 < c < 16$, at which $\displaystyle\int_0^{c} P(x)\,dx = 400$. **[2 marks]**

Worked Solutions

Q1 — Antiderivative of a polynomial [EASY]

Integrate each term separately by increasing the power by one and dividing by the new power, then attach the constant of integration.

$$\int (3x^2 + 4x - 5)\,dx = \frac{3x^3}{3} + \frac{4x^2}{2} - 5x + c$$

M1

$$= x^3 + 2x^2 - 5x + c$$

A1

Warning: omitting $+c$ from an indefinite integral loses the final accuracy mark. **A1**

Q2 — Integrating a negative power [EASY]

Rewrite $\frac{2}{x^3} = 2x^{-3}$ so that the power rule can be applied to both terms.

$$\int (6x^2 - 2x^{-3})\,dx$$

M1

Increase each power by one and divide by the new power:

$$= \frac{6x^3}{3} - \frac{2x^{-2}}{-2} + c = 2x^3 + x^{-2} + c$$

A1

$$= 2x^3 + \frac{1}{x^2} + c$$

A1

Warning: omitting $+c$ from an indefinite integral loses the final accuracy mark. Also take care with the sign when integrating a negative power: dividing by -2 turns the minus sign positive.

Q3 — Finding the constant of integration [EASY]

Integrate $f'(x) = 4x - 3$ to find the general form of $f(x)$, including $+c$.

$$f(x) = \int (4x - 3)\,dx = 2x^2 - 3x + c$$

M1

Substitute the known point $(2, 5)$ to find c: since $f(2) = 5$,

$$2(2)^2 - 3(2) + c = 5 \implies 8 - 6 + c = 5 \implies c = 3$$

M1

$$f(x) = 2x^2 - 3x + 3$$

A1

Warning: if $+c$ is omitted during the integration step, it is impossible to apply the given point, so the method mark for substitution is also at risk.

Q4 — Integrating positive and negative powers [EASY]

Apply the power rule to each term; increase each exponent by one and divide by the new exponent.

$$\int (5x^4 + 2x^{-2})\,dx$$

M1

$$= \frac{5x^5}{5} + \frac{2x^{-1}}{-1} + c = x^5 - 2x^{-1} + c$$

A1

$$= x^5 - \frac{2}{x} + c$$

A1

Warning: omitting $+c$ from an indefinite integral loses the final accuracy mark.

Q5 — GDC area under a curve [EASY]

Write down the definite integral representing the shaded area before evaluating it on the GDC.

$$\text{Area} = \int_1^3 (2x^3 - 3x + 1)\,dx$$

M1

GDC: Math $\rightarrow$ Calculus $\rightarrow$ $\int f(x)\,dx$, enter $2x^3 - 3x + 1$, lower limit 1, upper limit 3.
Unrounded value: 22.0

$$\text{Area} = 22.0 \text{ square units}$$

A2

Warning: the function stays positive on $[1, 3]$, so no sign-split is needed; always sketch or check values before deciding whether to split the interval.

Q6 — Trapezoidal rule with four strips [EASY]

State the strip width: the interval $[1, 5]$ is divided into 4 strips, so $h = \dfrac{5-1}{4} = 1$. There are 5 table values for 4 strips.

M1

Apply the trapezoidal rule with ends $g(1) = 3.0$ and $g(5) = 4.5$, and middles $g(2) = 4.2$, $g(3) = 6.1$, $g(4) = 5.8$:

$$\int_1^5 g(x)\,dx \approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)] = \frac{1}{2}[3.0 + 4.5 + 2(4.2 + 6.1 + 5.8)]$$

M1

$$= \frac{1}{2}[7.5 + 2(16.1)] = \frac{1}{2}(7.5 + 32.2) = \frac{1}{2}(39.7) = 19.85$$

$$\approx 19.9 \text{ (3 s.f.)}$$

A1

Warning: 4 strips require 5 values ($x = 1, 2, 3, 4, 5$); using only 4 values is the classic strips-vs-values error.

Q7 — Definite integral by GDC [EASY]

Write the definite integral in full before using the GDC to evaluate it.

$$\int_0^3 (x^2 + 2)\,dx$$

M1

GDC: Math $\to$ Calculus $\to$ $\int f(x)\,dx$, enter $x^2 + 2$, lower limit 0, upper limit 3.
Unrounded value: 15.0

$$= 15.0$$

A2

Warning: always write the definite integral with correct limits before pressing the GDC key; swapping the limits gives the negative of the answer.

Q8 — Finding f(0) from f' and a point [EASY]

Integrate $f'(x) = 6x^2 - 4x + 1$ to obtain the general expression for $f(x)$ with constant c.

$$f(x) = \int (6x^2 - 4x + 1)\,dx = 2x^3 - 2x^2 + x + c$$

M1

Substitute the known point $(1,3)$ to find c: since $f(1)=3$,

$$2(1)^3 - 2(1)^2 + 1 + c = 3 \implies 2 - 2 + 1 + c = 3 \implies c = 2$$

M1

Evaluate $f(0)$:

$$f(0) = 2(0)^3 - 2(0)^2 + 0 + 2 = 2$$

A1

Warning: omitting $+c$ makes it impossible to use the given point, so carry c through from the very first line of integration.

Q9 — Antiderivative of a cubic polynomial [EASY]

Integrate each term separately by increasing the power by one and dividing by the new power, then attach the constant of integration.

$$\int (8x^3 - 3x^2 + 7)\,dx$$

M1

$$= \frac{8x^4}{4} - \frac{3x^3}{3} + 7x + c = 2x^4 - x^3 + 7x + c$$

A1
A1

Warning: omitting $+c$ from an indefinite integral loses the final accuracy mark; every indefinite integral must include the arbitrary constant.

Q10 — GDC area under a quadratic curve [EASY]

Write the definite integral representing the shaded area before using the GDC to evaluate it.

$$\text{Area} = \int_0^4 (3x^2 + 2)\,dx$$

M1

GDC: Math $\to$ Calculus $\to \int f(x)\,dx$, enter $3x^2+2$, lower limit 0, upper limit 4.
Unrounded value: 72.0

$$\text{Area} = 72.0 \text{ square units}$$

A2

Warning: the function $h(x) = 3x^2 + 2$ is always positive on $[0,4]$, so the GDC result gives the area directly without any sign adjustment needed.

Q11 — Finding the constant of integration in a growth context [MEDIUM]

(a) Integrate $\frac{dN}{dt}$ term by term with respect to t to recover $N(t)$, remembering to include the constant of integration.

$$N(t) = \int (3t^2 - 4t + 5)\, dt = t^3 - 2t^2 + 5t + c$$

M1

Substitute the initial condition $N(1) = 6$ to find c:

$$1 - 2 + 5 + c = 6 \implies c = 2$$

$$N(t) = t^3 - 2t^2 + 5t + 2$$

A1A1

(b) Substitute $t = 3$ into the expression found in part (a):

$$N(3) = 27 - 18 + 15 + 2 = 26 \text{ thousand bacteria}$$

A1

Warning: Omitting $+c$ before applying the condition loses both the M1 and the follow-through A1; the answer $c = 2$ is only valid when $+c$ is written in the integrated line.

Q12 — Trapezoidal rule for a river cross-section [MEDIUM]

Identify $h = 2$ (the strip width, equal to the spacing between measurements) and note that 6 values give exactly 5 strips. **M1**

Apply the trapezoidal rule with the bracket showing ends and middles:

$$A \approx \frac{h}{2}[(d_0 + d_5) + 2(d_1 + d_2 + d_3 + d_4)] = \frac{2}{2}[(0 + 0) + 2(1.4 + 2.3 + 2.8 + 1.9)]$$

M1

$$= 1 \times [0 + 2(8.4)] = 1 \times 16.8 = 16.8\,\text{m}^2$$

A1

The estimate is an underestimate because the curve of the river's cross-section is concave down (bends above its chords), so each trapezoid lies below the true profile. **R1**

Warning: The most common trap is counting 6 strips instead of 5; the table has 6 *values* but only 5 *strips* (intervals) between them. Always check: number of strips $=$ number of values -1.

Q13 — Integrating a negative power; finding the constant [MEDIUM]

(a) Rewrite $\dfrac{4}{x^3} = 4x^{-3}$ before integrating, to apply the power rule correctly.

$$f(x) = \int (6x^2 - 4x^{-3})\,dx = 2x^3 - \frac{4x^{-2}}{-2} + c = 2x^3 + 2x^{-2} + c$$

M1A1

Substitute the point $(1,\ 9)$ to find c:

$$f(1) = 2(1)^3 + 2(1)^{-2} + c = 2 + 2 + c = 9 \implies c = 5$$

$$f(x) = 2x^3 + 2x^{-2} + 5$$

A1

(b) Evaluate at $x = 2$:

$$f(2) = 2(8) + 2(0.25) + 5 = 16 + 0.5 + 5 = 21.5$$

A1

Warning: A sign error when integrating $-4x^{-3}$ is the key trap: dividing by the new power -2 produces a positive term $+2x^{-2}$. Write the division step explicitly to avoid the error.

Q14 — Trapezoidal rule, over/under from concavity [MEDIUM]

(a) With 4 strips on $[1, 4]$, the strip width is $h = \dfrac{4-1}{4} = 0.75$, giving x-values $1,\ 1.75,\ 2.5,\ 3.25,\ 4.$
Evaluate $g(x) = x^3 - 5x + 2$ at each x-value: **M1**

x	1	1.75	2.5	3.25	4
$g(x)$	-2	-1.546875	5.125	16.078125	50

$$\int_1^4 g(x)\,dx \approx \frac{0.75}{2}[(-2+50) + 2(-1.546875 + 5.125 + 16.078125)]$$

$$= 0.375[48 + 2(19.65625)] = 0.375[48 + 39.3125] = 0.375 \times 87.3125 = 32.74\ldots \approx 32.7$$

A1

(b) The curve $g(x) = x^3 - 5x + 2$ is convex (concave upward) on most of $[1, 4]$, so the trapezoidal chords lie *above* the curve; the estimate is an **overestimate**. **R1**

Warning: Four strips require five table values; writing only four values is a common miscount that invalidates the calculation.

Q15 — Definite integral for distance; domain validity [MEDIUM]

(a) The total distance travelled is the definite integral of $v(t)$ over $[0, 4]$ (since $v(t) \geq 0$ on this interval):

$$\text{Distance} = \int_0^4 (4t - t^2)\, dt$$

A1

(b) GDC: Graph/Calculate $\to \int f(x)\, dx$, enter $4t - t^2$, lower $= 0$, upper $= 4$.
The unrounded GDC output is $\frac{32}{3} = 10.666\ldots$

$$\text{Distance} = 10.7\,\text{m} \quad (3\,\text{s.f.})$$

A2

(c) For $t > 4$ the model gives $v(t) < 0$, meaning the car would travel in reverse indefinitely, which is not physically realistic for a car that has already stopped. **A1**

Warning: Check that $v(t) \geq 0$ on $[0, 4]$ before claiming the integral equals distance; $4t - t^2 = t(4 - t) \geq 0$ on $[0, 4]$, so no sign-split is needed here. If v were negative on part of the interval, the area would need to be handled as $\int |v|\, dt$.

Q16 — Antiderivative and constant of integration from a point [MEDIUM]

(a) Integrate $\frac{dy}{dx}$ term by term with respect to x:

$$y = \int (5x^4 - 3x^2 + 2)\, dx = x^5 - x^3 + 2x + c$$

M1A1

Substitute the point $(2, 30)$ to determine c:

$$30 = (2)^5 - (2)^3 + 2(2) + c = 32 - 8 + 4 + c = 28 + c \implies c = 2$$

$$y = x^5 - x^3 + 2x + 2$$

A1

(b) The y-intercept is the value of y when $x = 0$:

$$y(0) = 0 - 0 + 0 + 2 = 2$$

A1

Warning: Forgetting $+c$ before substituting the given point makes it impossible to satisfy the condition; this is the most penalised omission in this archetype.

Q17 — GDC area vs trapezoidal estimate for a square-root function [MEDIUM]

(a) Write the integral before using the GDC:

$$\text{Area} = \int_0^8 \sqrt{x+1}\,dx$$

GDC: Graph $\to \int f(x)\,dx$, enter $\sqrt{x+1}$, lower $= 0$, upper $= 8$.
Unrounded value: $\frac{52}{3} = 17.333\ldots$

$$\text{Area} = 17.3\,\text{m}^2 \quad (3\,\text{s.f.})$$

A2

(b) Four strips on $[0, 8]$ give $h = 2$, with x-values $0, 2, 4, 6, 8$ (5 values for 4 strips). **M1**

x	0	2	4	6	8
$\sqrt{x+1}$	1	$\sqrt{3} \approx 1.732$	$\sqrt{5} \approx 2.236$	$\sqrt{7} \approx 2.646$	3

$$\approx \frac{2}{2}[(1+3) + 2(1.732 + 2.236 + 2.646)] = 1 \times [4 + 2(6.614)] = 4 + 13.228 = 17.2\ (3\text{ s.f.})$$

A1

Warning: Four strips require *five* x-values; a table with only four values is the strips-vs-values trap and will give an incorrect application of the rule.

Q18 — Antiderivative for cost function; constant from fixed cost [MEDIUM]

(a) The total cost function is the antiderivative of the marginal cost:

$$C(q) = \int (0.6q^2 - 8q + 40)\,dq = 0.2q^3 - 4q^2 + 40q + c$$

M1A1

Apply the fixed cost condition $C(0) = 200$ to find c:

$$200 = 0 - 0 + 0 + c \implies c = 200$$

$$C(q) = 0.2q^3 - 4q^2 + 40q + 200$$

A1

(b) Substitute $q = 10$:

$$C(10) = 0.2(1000) - 4(100) + 40(10) + 200 = 200 - 400 + 400 + 200 = \$400$$

A1

Warning: The constant c represents the fixed cost and equals 200 here; setting $c = 0$ is a common error that loses the A1 for the constant and produces an incorrect answer in part (b).

Q19 — Integrating acceleration to find velocity; constant from initial condition [MEDIUM]

(a) Velocity is the antiderivative of acceleration, so integrate $a(t)$ with respect to t:

$$v(t) = \int (6t - 4)\,dt = 3t^2 - 4t + c$$

M1A1

Apply the initial condition $v(0) = 7$ to find c:

$$7 = 3(0)^2 - 4(0) + c \implies c = 7$$

$$v(t) = 3t^2 - 4t + 7$$

A1

(b) Substitute $t = 5$ into the expression found in part (a):

$$v(5) = 3(25) - 4(5) + 7 = 75 - 20 + 7 = 62\,\text{m s}^{-1}$$

A1

Warning: Omitting $+c$ before applying the initial condition is the key trap; without the constant, the step $c = 7$ cannot be shown and the function found in (a) is incorrect.

Q20 — GDC area with sign-split for a curve crossing the x-axis [MEDIUM]

(a) Find the x-intercepts by solving $x^2-4x+3 = 0$: $(x-1)(x-3) = 0$, so $x = 1$ and $x = 3$. Sketch shows a parabola opening upward, crossing the x-axis at $x = 1$ and $x = 3$, dipping below the axis between them.
A1

(b) On $[0, 1]$ the curve is above the x-axis, and on $[1, 3]$ it is below; the total area requires splitting the integral at $x = 1$. **M1**

GDC: Compute $\int_0^1 (x^2 - 4x + 3)\,dx$ and $\int_1^3 (x^2 - 4x + 3)\,dx$ separately.

GDC: Graph $\rightarrow \int f(x)\,dx$, enter $x^2 - 4x + 3$:

$$\int_0^1 (x^2 - 4x + 3)\,dx = \tfrac{4}{3} = 1.333\ldots, \qquad \int_1^3 (x^2 - 4x + 3)\,dx = -\tfrac{4}{3} = -1.333\ldots$$

A1

Total area $= \left|\tfrac{4}{3}\right| + \left|-\tfrac{4}{3}\right| = \tfrac{4}{3} + \tfrac{4}{3} = \tfrac{8}{3} = 2.67$ (3 s.f.) **A1**

Warning: Evaluating $\int_0^3 (x^2 - 4x + 3)\,dx$ as a single GDC computation gives $\tfrac{4}{3} - \tfrac{4}{3} = 0$, which is incorrect for area; whenever the curve crosses the x-axis inside the interval, the integral must be split and each piece taken as an absolute value.

Q21 — Trapezoidal rule with unknown strip height and percentage error [HARD]

(a) The trapezoidal rule with $h = 1$ and 6 strips gives $\frac{h}{2}(y_0+2y_1+2y_2+2y_3+2y_4+2y_5+y_6)$; substituting the known values and equating to 10.1 lets us find k. **M1**

$$\frac{1}{2}(0 + 2(1.4) + 2(2.3) + 2(2.8) + 2k + 2(1.6) + 0) = 10.1$$

$$\frac{1}{2}(2.8 + 4.6 + 5.6 + 2k + 3.2) = 10.1$$

$$\frac{1}{2}(16.2 + 2k) = 10.1 \implies 16.2 + 2k = 20.2 \implies k = 2.1 \quad \textbf{AG}$$

Warning: Do not confuse 6 strips (requiring 7 table values) with 6 table values; the 6-strip formula uses all seven ordinates.
(b) The curve bends *downward* (concave) across the river, so the straight-line chords of each trapezoid lie **below** the curve. **R1**
Therefore the trapezoidal rule gives an **underestimate**. **A1**
(c) Percentage error $= \dfrac{|10.1 - 10.44|}{10.44} \times 100 = \dfrac{0.34}{10.44} \times 100 = 3.257\ldots\% \approx 3.26\%$. **A1**
Warning: Always divide by the *true* (GDC) value, not the estimate.

Q22 — Antiderivative with initial condition and zero-search [HARD]

(a) Integrate $v(t)$ term by term to find the general antiderivative, then apply $s(0) = 5$ to find c. **M1**

$$s(t) = \int (3t^2 - 8t + 4)\,dt = t^3 - 4t^2 + 4t + c$$

Substituting $s(0) = 5$: $0 - 0 + 0 + c = 5 \implies c = 5$. **A1**

$$s(t) = t^3 - 4t^2 + 4t + 5\textbf{A1}$$

Warning: Omitting $+c$ before applying the initial condition is the most common error here; the constant of integration is non-negotiable.
(b) GDC: substitute $t = 4$: $s(4) = 64 - 64 + 16 + 5 = 21$ m. **A1**
(c) GDC: Equation solver, $t^3 - 4t^2 + 4t + 5 = 5 \implies t^3 - 4t^2 + 4t = 0 \implies t(t-2)^2 = 0$; the only solution in $0 < t \leq 4$ is $t = 2$ (the root $t = 0$ is excluded). **A1**

Q23 — Antiderivative of negative power, recover f from f' and a point [HARD]

(a) Integrate $f'(x)$ term by term. Note $6x^{-2}$ integrates to $-6x^{-1}$, not $-6x^{-3}$; verify the sign carefully. **M1**

$$f(x) = \int (6x^{-2} - 4x + 3)\,dx = -6x^{-1} - 2x^2 + 3x + c$$

Apply $f(1) = 9$: $-6 - 2 + 3 + c = 9 \implies c = 14$. **A1**

$$f(x) = -\frac{6}{x} - 2x^2 + 3x + 14 \textbf{A1}$$

Warning: A sign error when integrating $6x^{-2} \to -6x^{-1}$ (not $+6x^{-1}$) is the main trap; the power rule gives $\frac{6}{-1}x^{-1}$.
(b) $f(3) = -\frac{6}{3} - 2(9) + 3(3) + 14 = -2 - 18 + 9 + 14 = 3$. **A1**
(c) The term $-6x^{-1} = -6/x$ is undefined at $x = 0$, so the model has no meaning for $x \le 0$. **A1**

Q24 — AG derivative then GDC area between curve and horizontal line [HARD]

(a) Differentiate $g(x) = x^3 - 6x^2 + 9x + 2$ term by term. **M1**

$$g'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3) \quad \textbf{AG}$$

The factorisation must be shown explicitly for the method mark to be awarded.
(b) The enclosed region lies between $y = g(x)$ and $y = 2$. From (a), $g(1) = 1 - 6 + 9 + 2 = 6$ and $g(3) = 27 - 54 + 27 + 2 = 2$, confirming the region is above $y = 2$ on $[1, 3]$. Set up the definite integral of the *difference*. **M1**

$$\text{Area} = \int_1^3 [g(x) - 2]\, dx = \int_1^3 (x^3 - 6x^2 + 9x)\, dx$$

GDC: `Integral, lower = 1, upper = 3`, expression $= x^3 - 6x^2 + 9x$. **(M1)**
Unrounded: $4.000\ldots$

$$\text{Area} = 4 \text{ square units} \textbf{A1}$$

Warning: Integrating $g(x)$ alone (not subtracting $y = 2$) gives ≈ 10, which is incorrect. Always subtract the lower boundary function.

Q25 — Trapezoidal rule vs GDC integral with concavity justification [HARD]

(a) With 4 equal strips on $[0, 4]$: $h = \frac{4-0}{4} = 1$. Evaluate $I(x)$ at $x = 0, 1, 2, 3, 4$. **M1**
$I(0) = 80,\ I(1) = 89,\ I(2) = 92,\ I(3) = 89,\ I(4) = 80$.

$$\text{Estimate} = \frac{1}{2}(80 + 2(89) + 2(92) + 2(89) + 80) = \frac{1}{2}(700) = 350 \textbf{A1}$$

Units: the integral has units lux·metres, but the numerical answer is 350. **A1**
Warning: 4 strips require 5 ordinates ($x = 0, 1, 2, 3, 4$); using only 4 values misses one strip.
(b) GDC: `Integral, lower = 0, upper = 4`, expression $= 80 + 12x - 3x^2$.
Unrounded: $349.333\ldots \approx 349$ (3 s.f.). **A1**
(c) $I''(x) = -6 < 0$ for all x, so the curve is concave (bends downward); the trapezoid chords lie *below* the curve. **R1**
Therefore the trapezoidal estimate of 350 is an **overestimate** of the true value ≈ 349.

Q26 — Antiderivative with initial condition, temperature model and maximum [HARD]

(a) Integrate $\frac{dT}{dt}$ term by term and apply the condition $T(2) = 18$. **M1**

$$T(t) = \int (0.4t - 0.05t^2)\,dt = 0.2t^2 - \frac{0.05}{3}t^3 + c$$

Applying $T(2) = 18$: $0.2(4) - \frac{0.05}{3}(8) + c = 18 \implies 0.8 - 0.1\overline{3} + c = 18 \implies c = 17.3\overline{3}$. **A1**

$$T(t) = 0.2t^2 - \frac{t^3}{60} + \frac{52}{3}\textbf{A1}$$

Warning: Premature rounding of c here propagates to part (b); keep the exact fraction $c = 52/3$.
(b) $T(12) = 0.2(144) - \frac{1728}{60} + \frac{52}{3} = 28.8 - 28.8 + 17.3\overline{3} = 17.\overline{3} \approx 17.3\,^\circ$C. **A1**
(c) Set $\frac{dT}{dt} = 0$: $0.4t - 0.05t^2 = 0 \implies t(0.4 - 0.05t) = 0 \implies t = 8$.
Since $\frac{dT}{dt} > 0$ for $0 < t < 8$ and < 0 for $8 < t \leq 12$, the temperature is greatest at $t = 8$ hours. **A1**

Q27 — AG definite integral then reverse: solve for parameter a [HARD]

(a) Find the antiderivative of $h'(x) = 5x^{-3} + 2x$, noting $5x^{-3}$ integrates to $\frac{5x^{-2}}{-2} = -\frac{5}{2x^2}$. **M1**

$$\int_1^a (5x^{-3} + 2x)\,dx = \left[-\frac{5}{2x^2} + x^2\right]_1^a$$

$$= \left(-\frac{5}{2a^2} + a^2\right) - \left(-\frac{5}{2} + 1\right) = a^2 - \frac{5}{2a^2} + \frac{5}{2} - 1 = a^2 - \frac{5}{2a^2} + \frac{3}{2} \quad \textbf{AGA1}$$

Unrounded intermediate: $\left(-\frac{5}{2} + 1\right) = -\frac{3}{2}$, so subtracting gives $+\frac{3}{2}$. **A1**
Warning: The sign error $5x^{-3} \to +\frac{5}{2x^2}$ (forgetting the negative from the power rule) is the most common trap.
(b) Set the expression equal to 20 and solve for a using a GDC. **M1**

$$a^2 - \frac{5}{2a^2} + \frac{3}{2} = 20$$

GDC: `Equation solver`, expression $= a^2 - 5/(2a^2) + 1.5 - 20$, solve for $a > 1$.
Unrounded: $a = 4.3253\ldots \approx 4.33$ (3 s.f.). **A1**

Q28 — Trapezoidal rule on irregular data and percentage error [HARD]

(a) The 5 strips each have width $h = 1.5$. There are 6 ordinates at $x = 0, 1.5, 3, 4.5, 6, 7.5$. **M1**

$$\text{Estimate} = \frac{1.5}{2}(0 + 2(1.2) + 2(2.1) + 2(2.4) + 2(1.8) + 0)$$

$$= 0.75(0 + 2.4 + 4.2 + 4.8 + 3.6 + 0) = 0.75 \times 15 = 11.25\ \text{m}^2\textbf{A1}$$

State units: 11.25 m^2. **A1**

Warning: 5 strips on $[0, 7.5]$ give $h = 7.5/5 = 1.5$, requiring 6 table values. Using $h = 1$ (the spacing between integers) ignores the actual data spacing and is wrong.

(b) Percentage error $= \dfrac{|11.25 - 9.23|}{9.23} \times 100$.

Unrounded: $\dfrac{2.02}{9.23} \times 100 = 21.884\ldots\%$. **(M1)**

$$\approx 21.9\%\textbf{A1}$$

Warning: Divide by the *true* value (9.23), not the estimate (11.25).

Q29 — Two-condition parameter hunt becoming simultaneous equations [HARD]

(a) Integrate $f(x) = px^2 + qx$ and apply the definite integral condition. **M1**

$$\int_0^2 (px^2 + qx)\,dx = \left[\frac{p}{3}x^3 + \frac{q}{2}x^2\right]_0^2 = \frac{8p}{3} + 2q = 10 \quad \textbf{AGA1}$$

(b) Differentiate: $f'(x) = 2px + q$. Apply $f'(1) = 7$: $2p + q = 7$. **M1**
Now solve the simultaneous system:

$$\frac{8p}{3} + 2q = 10 \quad \text{and} \quad 2p + q = 7$$

From the second equation $q = 7 - 2p$; substitute: $\frac{8p}{3} + 2(7 - 2p) = 10 \implies \frac{8p}{3} - 4p = -4 \implies -\frac{4p}{3} = -4 \implies p = 3$.
Then $q = 7 - 6 = 1$.

$$p = 3, \quad q = 1\textbf{A1}$$

Warning: Premature rounding of $8p/3$ before solving will give incorrect non-integer answers; keep exact fractions throughout.
(c) The area is the value of the integral already computed: since $p = 3$, $q = 1$, the integral equals **10** square units (given). **A1**
(Verify: $f(x) = 3x^2 + x \geq 0$ on $[0, 2]$, so no sign split is needed.)

Q30 — Antiderivative with fractional power, GDC definite integral, reverse threshold [HARD]

(a) Integrate term by term. Recall $\int x^{1/2}\,dx = \frac{2}{3}x^{3/2}$. **M1**

$$\int P(x)\,dx = \int (40 + 18x^{1/2} - 3x)\,dx = 40x + 18 \cdot \frac{2}{3}x^{3/2} - \frac{3}{2}x^2 + c = 40x + 12x^{3/2} - \frac{3}{2}x^2 + c\textbf{A1}$$

Warning: Omitting $+c$ on an indefinite integral costs a mark; it must appear on every indefinite integral.
(b) GDC: `Integral, lower = 0, upper = 16,` expression $= 40 + 18\sqrt{x} - 3x$.
Unrounded: $737.3\overline{3}\ldots$

$$\int_0^{16} P(x)\,dx = 737.3 \text{ (4 s.f.)} \textbf{A1}$$

(c) Use the antiderivative from (a) with lower limit 0 to set up $\int_0^c P(x)\,dx = 400$. **M1**

$$\left[40x + 12x^{3/2} - \frac{3}{2}x^2\right]_0^c = 40c + 12c^{3/2} - \frac{3}{2}c^2 = 400$$

GDC: `Equation solver`, expression $= 40c + 12c^{3/2} - 1.5c^2 - 400$, solve for c in $(0, 16)$.
Unrounded: $c = 7.1589\ldots \approx 7.16$ (3 s.f.). **A1**
Warning: There may be two numerical solutions; verify $0 < c < 16$ and check both roots if the solver suggests a second one outside the domain.